Task: measuring arm span
   Collect data.

Organize your data.
Look for ways to describe the data.
Think of other ways to represent the data.
Why is it important to order numbers in a data set?

Range

Mean

Frequency Tables, Dot plot, Histogram

*Circle the mean on the number line of your dot plot, histograms.*
Practice.

The data shown represents the number of miles per gallon that 30 selected four-wheeler drive vehicles obtained in city driving.

Organize the data.

<table>
<thead>
<tr>
<th>12</th>
<th>17</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>18</td>
<td>12</td>
<td>16</td>
<td>17</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>12</td>
<td>15</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
<td>15</td>
<td>12</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>19</td>
<td>13</td>
<td>16</td>
<td>18</td>
<td>16</td>
<td>14</td>
</tr>
</tbody>
</table>


Identify the minimum, maximum, and calculate the range and mean.

Organizing data in a frequency table.

Construct a dot plot of the data using appropriate scale.

Create a histogram of the data using appropriate scale.

Write at least two sentences to describe the data.
Write a sentence to compare dot plots and histograms. (How are they similar and how are they different?)

Practice.

Given the frequency table, create a histogram.

The air quality measured for selected U.S. cities in 2002 is shown below. The data are the number of days per year that the cities failed to meet the acceptable standards.

<table>
<thead>
<tr>
<th># of Days</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-27</td>
<td>19</td>
</tr>
<tr>
<td>28-55</td>
<td>6</td>
</tr>
<tr>
<td>56-83</td>
<td>2</td>
</tr>
<tr>
<td>84-111</td>
<td>0</td>
</tr>
<tr>
<td>112-139</td>
<td>0</td>
</tr>
<tr>
<td>140-167</td>
<td>3</td>
</tr>
<tr>
<td>168-195</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: *World Almanac and Book of Facts*

Use your histogram to describe the data.
Practice Set 1.

1. The acreage of ten U.S. National Parks is shown below (in thousands of acres).

   a. Organize the data.

   Source: The Universal Almanac

   b. Identify the minimum and maximum values. Find the range.

   c. Find the mean of the data set.

   d. Create a dot plot for the data. Use appropriate labels and scale.

2. Collect a data set of at least 10 values. (For examples, find prices of ten different foods at the grocery store or measure the length of ten different blades of grass).

   a. Clearly describe what the data is.

   b. Organize the data.

   c. Identify the minimum and maximum. Calculate the mean.

   d. Construct a histogram or dot plot for your data. Use appropriate labels and scale.

3. ACT Practice. Organizing data in a frequency table Ralph calculated the mean number of pages in 8 books as 224. He later realized that he used 220 pages for a book that actually had 200 pages. What is the correct mean number of pages in the 8 books?
ACT Warm-Up

Evaluate. \[2(1-5)+3\]

Evaluate. \[200-3(\sqrt{9}-4)^2\]

What do you know about the median of a set of data?

What do you need to do to find the median of a set of data?

Task: car prices data.
A list of ten 2011 car prices is given below.

<table>
<thead>
<tr>
<th>$22,000</th>
<th>$40,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$33,000</td>
<td>$39,000</td>
</tr>
<tr>
<td>$23,000</td>
<td>$27,000</td>
</tr>
<tr>
<td>$44,000</td>
<td>$28,000</td>
</tr>
<tr>
<td>$34,000</td>
<td>$380,000</td>
</tr>
</tbody>
</table>

Organize the data.
Use the range, mean, and median to describe the data.
Think of other ways to represent the data.
How is the median different from the mean?

What do you think a Quartile does?

Box plot.

Data Collection

Collect the day of the month that you were born and data from your classmates. For example, if you were born on June 17th, use a 17. Organize as you collect.

Calculate the median.

Calculate the Quartiles.

Construct a Box Plot.

Calculate the InterQuartile range (IQR).
Why do we have histograms and box plots?

Which value, the mean or median, is a better description of the center of the data? Why?

Outlier

Refer to car price data set. Create a box plot. For this data set.

Discuss how far is too far?

John Tukey

Create a new box plot for the data above that identifies the outlier (not included in data set).

Write two sentences to compare the two box plots (one before and one after identifying the outlier).
Practice.

Construct a box plot for the number of bills enacted by Congress in the last several years.

88 245 153 241 170 410 136 241 198

Source: *USA Today*

Data Collection.

Student volunteers needed for push-up contest. Collect the number of push-ups done in one minute. (Use the data below if collection impossible).

12 16 21 17 23 13 23 24 35
15 25 25 36 38 14 26 30 40

Construct a histogram and a box plot for the data.

Use your histogram, box plot, and appropriate vocabulary to describe your data.
Practice Set 2. 

Name ___________________________

1. What do you need to make a box plot?

2. Describe how to make a histogram.

3. The annual number of deaths from tornadoes in the U.S. from 1990 to 2000 are given below.

Source: NOAA
Construct a box plot.

53 39 39 33 69 30 25 67 130 94 40

Complete the frequency table.

<table>
<thead>
<tr>
<th># of Deaths</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-34</td>
<td></td>
</tr>
<tr>
<td>35-44</td>
<td></td>
</tr>
<tr>
<td>45-54</td>
<td></td>
</tr>
<tr>
<td>55-64</td>
<td></td>
</tr>
<tr>
<td>65-74</td>
<td></td>
</tr>
<tr>
<td>75-84</td>
<td></td>
</tr>
<tr>
<td>85-94</td>
<td></td>
</tr>
<tr>
<td>95-104</td>
<td></td>
</tr>
<tr>
<td>105-114</td>
<td></td>
</tr>
<tr>
<td>115-124</td>
<td></td>
</tr>
<tr>
<td>125-134</td>
<td></td>
</tr>
</tbody>
</table>

Create a histogram using the frequency table.

4. Which graph most effectively represents the data? Why?

5. ACT Practice. What is the positive difference between the mean and the median of the 6 numbers given below?  

13  23  12  20  13  15
ACT Warm-Up

This graph shows the number of books on each of the 8 shelves of a bookcase.

Describe when the mean is more appropriate than the median to describe the center of data.

Describe when the median is a more appropriate measure of the center of data.

3 Histogram Examples to Explore

A. Describe the shape of the data.

Source: ideal.stat.wvu.edu

B. Describe the shape of the data.

Source: ritme.com

C. Describe the shape of the data.

Source: (adapted from Ocean copyright data supplied by the Met Office)
A. Describe **symmetric** data.

B. Describe data skewed to the left.

C. Describe data skewed to the right.

Practice.
Write a sentence for each histogram describing the shape. Use the vocabulary underlined above.
For symmetric data

Mean, \( \bar{X} \), as measure of center

Standard Deviation as a measure of spread

**Spread**

**Practice.** Find the mean, \( \bar{X} \), of the data set.

The annual number of children born in Pope County, IL for the last nine years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Children</th>
</tr>
</thead>
<tbody>
<tr>
<td>2019</td>
<td>29</td>
</tr>
<tr>
<td>2020</td>
<td>31</td>
</tr>
<tr>
<td>2021</td>
<td>34</td>
</tr>
<tr>
<td>2022</td>
<td>34</td>
</tr>
<tr>
<td>2023</td>
<td>28</td>
</tr>
<tr>
<td>2024</td>
<td>40</td>
</tr>
<tr>
<td>2025</td>
<td>32</td>
</tr>
<tr>
<td>2026</td>
<td>39</td>
</tr>
<tr>
<td>2027</td>
<td>32</td>
</tr>
</tbody>
</table>

Look at Calculating Standard Deviation for the practice data set.

Deviation:

\[ X - \bar{X} \]

Why are some deviations positive and some negative?

What would happen if you add all the deviations together?

What could we do to calculate an average deviation?

**Formula**

**Practice.**

1. Find the mean of the data set. Then find the standard deviation.

The costs of 10 electric smoothtop ranges rated very good or excellent by Consumer Reports in August 2002.

| Cost | 850  | 900  | 1400 | 1200 | 1050 | 750  |
Skewed data
Median as a measure of center

IQR as a measure of spread

Practice.
1. Find the median. Find the IQR. Draw a box plot to represent the data.
The costs of 10 electric smoothtop ranges rated very good or excellent by *Consumer Reports* in August 2002.
$850 900 1400 1200 1050 750

2. Compare the two box plots. Write three sentences to describe any similarities and differences.

3. Draw a box plot to represent the information given about what percentage of incoming freshmen usually graduate on time, in four years.

   Median 69.90%
   Minimum 43.20%
   Maximum 87.40%
   Lower Quartile (Q1) 59.15%
   Upper Quartile (Q3) 74.75%

Describe the data as symmetric or skewed and explain.

Based on your response, which is a better description of center and spread, the mean and standard deviation or the median and interquartile range? Why?
Compare histogram and box plot ... which is a better representation?

How will you decide which measure of center and spread you will use to describe a data set?
1. Write a sentence to describe the shape of the data. Include vocabulary from the lesson.

2. Find the mean of the data set. Then find the standard deviation.

   6 11 14 9 3 12 8

3. Compare the two box plots. Write three sentences to describe any similarities and differences.
4. The annual returns of Treasury bills from a 1950-2003 sampling, are given in percents. 

\[ 0.9 \quad 1.2 \quad 1.5 \quad 1.6 \quad 1.8 \quad 2.7 \quad 3.1 \quad 3.5 \quad 4.7 \quad 5.2 \]

\[ 6.4 \quad 7.2 \quad 10.4 \quad 11.3 \quad 14.7 \]

Given the data set, choose an appropriate measure of center and spread (mean and standard deviation or median and IQR). Explain your choice.

---

ACT practice

These are Grace’s scores on a series of tests: 78, 83, 85, 93, 87, 85, 90, 79, 95, and 85. If she scores 90 on the next test, which statement will be true?

A. The median score will increase, but the mean score will not change.

B. The mean score will increase, but the median score will not change.

C. The median score and the mean score both will increase.

D. The mean score will increase, but the median score will decrease.
Which expression is equivalent to $2x + 3y - 5x^2 - 10y + 7x^2$?

A. $4x^2 - 7y$
B. $2x^2 + 2x - 7y$
C. $2x^2 - 5xy$
D. $2x + 13y + 12x^2$

Calculation mean, standard deviation, and Quartiles using calculator. (Instructions for a TI-83 calculator).
1. Enter data into calculator (under L1).
   ![STAT ENTER CLEAR ENTER](image)
   Enter the data.

2. Go to the home screen.
   ![2nd MODE](image)

3. Run 1-variable statistics to calculate the mean, standard deviation, etc.
   ![STAT ENTER 2nd 1 ENTER](image)

4. Arrow down to see all the Quartiles.

Constructing a Histogram on your calculator.

1. Enter data into calculator.
   ![STAT ENTER CLEAR ENTER](image)

2. Constructing the histogram by turning on the stat plots.
   ![2nd Y= ENTER ENTER](image)

3. Now select the histogram plot.

4. Zoom Stats will graph the data and change your data at the same time.
Practice.
1. The level of various substances in the blood influences our health. Given are the measurements of the level of phosphate in the blood of a patient, in milligrams per deciliter of blood, made on 6 consecutive visits to a clinic.

   5.6  5.2  4.6  4.9  5.7  6.4

   Use your calculator to find the mean and standard deviation.

   Use your calculator to find the median and interquartile range. Use this information to draw a box-plot.

2. The Degree of Reading Power test is often used to measure reading ability of children. Given are DRP scores from eleven third grade students.

   40  26  39  14  42  18  25  43  46  27  19

   Use your calculator to find the mean and standard deviation.

   Use your calculator to find the median and interquartile range. Use this information to draw a box-plot.

3. The weights in grams of 10 loon chicks are given.

   79.5  87.5  88.5  89.2  91.6  84.5  82.1  85.7  82.3  89.3

   Use your calculator to find the mean and standard deviation.

   Use your calculator to find the median and interquartile range. Use this information to draw a box-plot.
Task: Collect the number of siblings for each class member.

Organize the data.
Represent the data in different formats.
Choose appropriate measures to describe the center and spread of the data.
1. Given the data for recent Algebra 2 test scores,
   72  70  77  76  90  68  81  86  34  94  71
   84  89  67  19  85  75  66  80  94

   Create a frequency table.

   Create a histogram.

   Create a box plot.

   Choose an appropriate measure of center and spread. Explain your choice.

ACT Problem.
What is the value of the expression
\[ \frac{a}{x} + \frac{a^2}{x^2} + \frac{a}{y} + \frac{a^2}{y^2} \] when \( x = 2, \ y = 2, \) and \( a = 2 \)?

A. 0  
B. 3  
C. 4  
D. 6
Ruth plotted 4 points on this graph.

Which point has a y-coordinate of -3?

Categorical data

Class data collection
Answer the following questions based on the data collected.
1. What percent of females are brown-eyed?

2. What percent of brown-eyed students are female?

3. What percent of all students are brown-eyed females?

4. Compare the percent who are female among the blue-eyed students to the percent of all students who are female.

5. What’s the conditional distribution of Eye color for the males?

Make additional notes or corrections based on class discussion.
Data from the *Titanic* has been placed into a two-way frequency table.

<table>
<thead>
<tr>
<th>Survival</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Crew</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alive</td>
<td>203</td>
<td>118</td>
<td>178</td>
<td>212</td>
<td>620</td>
</tr>
<tr>
<td>Dead</td>
<td>122</td>
<td>167</td>
<td>528</td>
<td>673</td>
<td>1480</td>
</tr>
<tr>
<td>Total</td>
<td>325</td>
<td>285</td>
<td>706</td>
<td>885</td>
<td>2106</td>
</tr>
</tbody>
</table>

As a group, fill in the missing totals. Calculate conditional distributions to determine if there was a relationship between the class of ticket a passenger held and the passenger’s chances of making it into the lifeboat.
Class data collection.

Does it seem that the two categories are related? Explain.
1. **PROM** Michael asks a random sample of 160 upperclassmen at his high school whether or not they plan to attend the prom. He finds that 44 seniors and 32 juniors plan to attend the prom, while 25 seniors and 59 juniors do not plan to attend. Organize the responses into a two-way frequency table.

2. Find the conditional distribution of attendance for juniors.

3. Would you expect distributions of these data sets to be symmetric or skewed? Explain your response.
   a. Players’ scores at the U.S. Open golf tournament in a given year.
   
   b. The length of the average hair on the heads of students in a class.
   
   c. The number of speeding tickets each student in the senior class of a college has ever had.
   
   d. Ages of people at a Little League baseball game.

4. Gather some data about family expenses. For example, how much is a monthly cable bill? We will use this data tomorrow.

**ACT Practice.**

**What is the value of the expression**

\[ 1 + \frac{1}{x} + \frac{x}{x^2} + \frac{x^2}{x^3} \quad \text{for} \quad x = -4 ? \]
ACT Warm-Up
\[
\frac{3}{4} \times \frac{2}{3} = ?
\]

A. \(2 \frac{2}{3}\)  
B. \(4 \frac{1}{2}\)  
C. \(4 \frac{5}{7}\)  
D. \(8 \frac{1}{6}\)  
E. None of the above

Information from Frequency tables (or other sources of information) in a Matrix
Write the examples below.

Quick Introduction

Notation
What are entries or elements of a matrix?

Dimensions of a Matrix

Write the dimensions of each matrix in the examples above.

What is the plural of matrix?
Household expenses

| water | power | cable | food | mortgage/rent |

Create and name matrices for the months of January, February, and March. Do you think the amounts will change?

A city accountant is interested in household expenses for the first quarter of each year. How could these totals be calculated?

Are there any restrictions on this operation?
To save time, the accountant decided to simply use January’s expenses and triple them to get first quarter totals. How could these totals be calculated?

Is a subtraction a possible matrix operation? If so, can you think of a possible application?

Are there any restrictions?
\[
\begin{bmatrix}
4 & 2 \\
\end{bmatrix} + \begin{bmatrix}
-2 & -6 \\
\end{bmatrix}
\]

\[-5\begin{bmatrix}
1 & -2 & -1 & 2 \\
\end{bmatrix}\]

\[-2u\begin{bmatrix}
7u & 3w^2 & 5u & 5 \\
\end{bmatrix}\]

\[
\begin{bmatrix}
-x - 1 & -2x & -5y \\
\end{bmatrix} - \begin{bmatrix}
y & -2 & -3x \\
\end{bmatrix}
\]
Consider the following example.
The elements of $A$ represent the number of three different parts in production at two factories. The elements of $B$ represent the labor hours required to produce each part at each of the two factories. What is the meaning of each element and how can we find $AB$? $BA$?

$$
A = \begin{bmatrix} 
40 & 30 & 80 \\
20 & 70 & 35 
\end{bmatrix} \quad B = \begin{bmatrix} 
4 & 3 \\
2 & 5 \\
6 & 2 
\end{bmatrix}
$$

Do $AB$ and $BA$ represent the same things?

Notes from class discussion.
Are there any restrictions on matrix multiplication?

Practice.

\[
\begin{bmatrix}-5 \\ 6 \\ 0\end{bmatrix} \cdot \begin{bmatrix}3 \\ -1\end{bmatrix}
\]

\[
\begin{bmatrix}-4 & -y \\ -2x & -4\end{bmatrix} \cdot \begin{bmatrix}-4x & 0 \\ 2y & -5\end{bmatrix}
\]

Write an example of a matrix multiplication that is undefined.
1. Write the dimensions of the following matrix.
   \[
   \begin{bmatrix}
   -1 & 1 & -1 \\
   5 & 2 & -5 \\
   6 & -5 & 1 \\
   -5 & 6 & 0 \\
   \end{bmatrix}
   \]

2. Simplify the following. If not possible, explain.
   \[
   \begin{bmatrix}
   z - 5 \\
   -6 \\
   -1 - 6z \\
   3y \\
   \end{bmatrix}
   + \begin{bmatrix}
   -3y \\
   3z \\
   5 + z \\
   4z \\
   \end{bmatrix}
   \]

   \[
   5\begin{bmatrix}
   6 & 1 & 2 & -6 \\
   \end{bmatrix}
   - \begin{bmatrix}
   1 & 6 & -6 & 6 \\
   \end{bmatrix}
   \]

   \[
   \begin{bmatrix}
   3 & -1 \\
   -3 & 6 \\
   -6 & -6 \\
   \end{bmatrix}
   \cdot \begin{bmatrix}
   -1 & 6 \\
   5 & 4 \\
   \end{bmatrix}
   \]

3. In the expression \( A \cdot B \), if \( A \) is a \( 3 \times 5 \) matrix, then what could be the dimensions of \( B \)?

4. The following data is the minimum, Q1, median, Q2, and maximum values found. Create a box plot for each data set. Write two sentences to compare the data sets.
   Male life expectancy in South American nations
   \[
   \begin{align*}
   58.0 & \quad 62.85 & \quad 68.65 & \quad 70.6 & \quad 72.4 \\
   \end{align*}
   \]
   Female life expectancy in South American nations
   \[
   \begin{align*}
   63.9 & \quad 70.95 & \quad 74.2 & \quad 77.1 & \quad 78.8 \\
   \end{align*}
   \]

   ACT Practice. \( 16 + 20 \div 4 - 2 \times 2 = ? \)
   
   A. 5 \hspace{0.2cm} B. 10
   C. 14 \hspace{0.2cm} D. 17
   E. 28
ACT Warm-Up

Which expression represents the perimeter of this triangle?

A. $8x^2 + 12x$
B. $4x^2 + 16x$
C. $4x^2 + 16x - 2$
D. $4x^2 - 8x - 2$

Toothpick figures

Try to determine the figure in Step 5.

Using different representations, try to explain how to find the figure in the next step.
What are some other attributes of the sequence that we could study?

Your assigned attribute:

Using multiple representations, describe the pattern.
Notes from other presentations
1. Focusing on perimeter of the figure, use multiple representations to determine the next step.

2. Given the frequency table of female SAT mathematics scores in the year 2000, Source: Collegeboard.org

Create a histogram to represent the data.

<table>
<thead>
<tr>
<th>Score</th>
<th>Percent of Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>200-299</td>
<td>3</td>
</tr>
<tr>
<td>300-399</td>
<td>14</td>
</tr>
<tr>
<td>400-499</td>
<td>33</td>
</tr>
<tr>
<td>500-599</td>
<td>31</td>
</tr>
<tr>
<td>600-699</td>
<td>15</td>
</tr>
<tr>
<td>700-800</td>
<td>4</td>
</tr>
</tbody>
</table>

**ACT Problem**

A 48 ft rope is cut into 5 pieces according to the ratio 2:2:3:4:5. What is the length, in feet, of the longest piece?  
A. 3  
B. 6  
C. 9  
D. 15
This diagram is used to classify shapes. In which region would a small shaded 5-sided figure be located?

A. Region A  
B. Region B  
C. Region C  
D. Region D

Task
Using different representations, try to explain how to find the figure in the next step.

Consider ways to find the 100th step.
Note from presentations
Recursive vs. Explicit

Function notation

Ex. From a picture sequence, Ruth noted the following.

<table>
<thead>
<tr>
<th>Step</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>…</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount</td>
<td>15</td>
<td>19</td>
<td>23</td>
<td>27</td>
<td>…</td>
<td>f(n)</td>
</tr>
</tbody>
</table>

Describe the sequence in words.

What is f(4) ?     How do you know?

What is f(7) ?

What is the meaning of f(n)?

What is the meaning of f(n-1)?

Give an example.

Write one sentence explaining how we can use function notation to describe sequences?

Additional ideas from class:

Ex. Now = Previous * 3
    Start with 4.

Use function notation to write this relationship.

Is this a recursive or explicit definition?
Ex. Bill says that to get the next term in a sequence, add 8 to the one before it. He also notes that the sequence starts with 5 dots.

Use function notation to write a recursive definition.

Try to determine an explicit definition.

Additional notes or corrections:

Ex. Amount at any step = 11 + 2 for each step

Use function notation to write this relationship.

Is this a recursive or explicit definition?

Ex. $f(n) = 14 + 5n$ What does this equation mean?

Is this a recursive or explicit definition?

Ex. $f(n) = f(n-1) + 9 \quad \text{What does this equation mean?}$

$f(1) = 10$

Is this a recursive or explicit definition?

Complete the sentence.
I think the difference between recursive and explicit definitions is
Consider the recursive definition: \( f(n) = f(n-1) + 5 \), and \( f(1) = 3 \). Fill in a table of values for this sequence.

<table>
<thead>
<tr>
<th>Step</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Consider \( f(n) = 3f(n-1) \), and \( f(1) = 4 \). Fill in a table of values for this sequence.

<table>
<thead>
<tr>
<th>Step</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Write one sentence to compare the two different sequences above.

Write any additional comments from class discussion.

Consider \( f(n) = 3 + 5(n-1) \). Fill in a table of values for this sequence.

<table>
<thead>
<tr>
<th>Step</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Write two sentences to describe the sequence above.

Write any additional comments from class discussion.
1. Draw the figure for Step 5 in the sequence.

Use different representations to describe how you came up with Step 5.

Consider ways to find the 100th step.

2. Write one sentence to compare recursive definitions and explicit definitions.

3. Given \( f(n) = f(n-1) + 8 \) and \( f(1) = 25 \), fill in a table of values for this sequence.

<table>
<thead>
<tr>
<th>Step</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

4. For the given data set, find the median and interquartile range. Construct a box-plot.

The following set of numbers are the amount of marbles fifteen different boys own (they are arranged from least to greatest).

18 27 34 52 54 59 61 68 78 82 85 87 91 93 100

ACT Practice.

What is the mean of 8, 7, 7, 5, 3, 2, and 2?
Using multiple representations, describe the pattern.
Notes from presentations
Cut and sort the following data tables that students collected from difference sequences.

<table>
<thead>
<tr>
<th>Step</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>54</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
</tr>
<tr>
<td>4</td>
<td>375</td>
</tr>
</tbody>
</table>

Tape or paste in your categories here.

<table>
<thead>
<tr>
<th>Step</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>54</td>
</tr>
</tbody>
</table>

Step Amount
1 4
2 12
3 24
4 40

Step Amount
1 2
2 12
3 24
4 40

Write one sentence to describe how your group sorted the data tables.
What are the sorting categories of the class?

Add the conventional names for these types of sequences.
1. If a table of values appears to have a common ratio or factor, what type of sequence does the table represent? Explain your response.

2. Is the equation \( f(n) = f(n-1) + 10 \) a recursive or explicit definition? Explain your response. Explain the meaning of the equation in terms of a sequence.

3. Use different representations to determine the next term in the sequence.

   Use function notation to write a recursive definition.

   Determine how you could find the figure for the 100th step.

   If possible, use function notation to write an explicit definition.

   Is this sequence arithmetic, geometric, or something else? Explain your response.

4. Use the following frequency table to create a histogram. Use appropriate labels and scale.

   Write a sentence to describe the shape of the data.

   In her garden, Ginny is creating a brick mosaic in a trapezoidal shape. The mosaic pattern has 6 rows. The first row has 8 bricks, and the last row has 24 bricks. Given that Ginny’s pattern follows an arithmetic sequence, how many bricks does she need?

   (Note: Partial bricks are allowed.)

   A. 112  B. 96  C. 64  D. 38
ACT Warm-Up

A total of $9,100 in prizes is awarded to the top 8 entries in a science project contest. The highest prize is $2,000. The differences between successive prizes are equal. What is the amount, in dollars, of the lowest prize?

A. $200  
B. $275  
C. $425  
D. $650

Write one sentence to compare an arithmetic and geometric sequence.

Add notes and/or corrections from class discussion.

Refer to problem 3 of Practice Set 7. The following is the same sequence with one difference.

![Diagram of sequence]

At the beginning  At one minute  At two minutes

How does this difference affect your work?

Scatter Plot

Write one sentence to explain any differences between this problem and that of problem 3 in the practice set.
Paper folding:

1. **Fold a sheet of paper in half as many times as possible.**
2. **After each fold, record three columns of information in a chart as illustrated below.**

<table>
<thead>
<tr>
<th>Number of Folds (n)</th>
<th>Number of Regions Formed on the Paper (R)</th>
<th>Area of Each Region Relative to the Whole Sheet of Paper (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>½ (The area of each region is half the area of the whole sheet.)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What if you don’t fold the paper? Could your record data in your table for that situation?

Use different representations and appropriate notation to describe your assigned sequence.

Would you describe this sequence as arithmetic, geometric, or neither? Explain.
Notes from other presentations.

Write several sentences comparing the sequence you studied to the sequence studied by your classmates.

Make additional notes after class discussion.
Write what you recall to compare arithmetic and geometric sequences.

Make corrections or add comments after a class discussion of a comparison of the two types of sequences.

Create your own table of values for an arithmetic sequence with increasing terms.

<table>
<thead>
<tr>
<th>Time</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Write the arithmetic sequence of your neighbor.

<table>
<thead>
<tr>
<th>Time</th>
<th>F(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Use function notation to write a recursive formula to describe their sequence.

Share your formula with your neighbor. Does your neighbor agree? Explain.

Create your own table of values for an arithmetic sequence with decreasing terms.

<table>
<thead>
<tr>
<th>Time</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Write the arithmetic sequence of your neighbor.

<table>
<thead>
<tr>
<th>Time</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Use function notation to write an explicit formula to describe their sequence.

Share your formula with your neighbor. Does your neighbor agree? Explain.
1. Create your own table of values for a geometric sequence with increasing terms.

<table>
<thead>
<tr>
<th>Time</th>
<th>( f(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Use function notation to write a recursive formula to describe this sequence.

Create a scatter plot to describe this sequence. Use appropriate scale and identify important aspects of the plot.

2. Write two sentences to describe how a geometric sequence with decreasing terms would be different from that of problem 1.

3. Describe \( f(n) = f(n-1) + 11 \) as an explicit or recursive definition. Explain your response.

4. Write two sentences to describe when you would choose to use the mean and standard deviation as a measure of center and spread instead of the median and IQR.

ACT Practice. Adam recorded the number of pairs of shoes sold in his shop in the first 4 months of the year in this table.

<table>
<thead>
<tr>
<th>Month</th>
<th>Number of pairs of shoes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>210</td>
</tr>
<tr>
<td>2</td>
<td>212</td>
</tr>
<tr>
<td>3</td>
<td>214</td>
</tr>
<tr>
<td>4</td>
<td>215</td>
</tr>
</tbody>
</table>

He found that the data followed an arithmetic sequence. Create a table that shows the number of shoes he should expect to sell in the last 4 months of the year, if the same pattern continues.
ACT Warm-Up

Carole wrote down the first few terms of an arithmetic sequence.

\[ 3, 4, 5, 6, 7, \ldots \]

What is the general term, \( f(n) \) of the sequence Carole wrote?

In your group, discuss the following two sequences. Use different representations to describe the tables of data.

1. (One representation may include a Scatter Plot to organize the sequence. Use appropriate scale and spacing.)

<table>
<thead>
<tr>
<th>Position</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
</tr>
</tbody>
</table>

If the sequence continues, will position 5 ever have a different term? Explain.

2.

<table>
<thead>
<tr>
<th>Position</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>48</td>
</tr>
</tbody>
</table>

If the sequence continues, will position 4 ever have a different term? Explain.
Write two sentences to compare the scatter plots you just constructed.

Write additional comments or corrections after class discussion.

Identify the sequence displayed in the following scatter plot. Use proper notation to describe the sequence.

If the sequence continues, will position 3 ever have a different term? Explain.

These sequences are examples of functions.

**Domain:** Input: position

- What are possible positions?

Write a sentence beginning with “The domain of the previous function is …”

**Range:** Output: terms

- For the last sequence, what are the possible terms?

Write a sentence beginning with “The range of the previous function is… “
As a group, consider the following sets. Determine whether each is a function or not. Explain.

1. A student’s name versus their school ID number.
2. A school ID number versus a student’s last name.
3. The cost of gas versus the amount of gas pumped.
4. The time of day with respect to the temperature.
5. The area of a circle depends on the radius.

### Table

<table>
<thead>
<tr>
<th>Step</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
</tbody>
</table>

6. Distance days

<table>
<thead>
<tr>
<th>distance</th>
<th>days</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

7. The area of a rectangle versus the height.

8. Diagram

9. The area of a rectangle versus the height.

Write additional notes or corrections based on class discussion.
Given the rule for a sequence \( f(n) = 5 \cdot 3^n \), create a table of values for the first six terms. Begin with \( n = 0 \).

<table>
<thead>
<tr>
<th>n</th>
<th>( f(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>135</td>
</tr>
<tr>
<td>4</td>
<td>405</td>
</tr>
<tr>
<td>5</td>
<td>1215</td>
</tr>
</tbody>
</table>

How would you describe this sequence/function?

Check with your neighbor. Do your descriptions agree? If necessary, make changes to your description to be precise.

For this function, what are the possible inputs?

Write one sentence beginning with “The domain of this function is …”.

For this function, what are the possible outputs?

Write one sentence beginning with “The range of this function is …”.

How many different outputs will a position of \( n = 3 \) produce? Explain.

Construct a Scatter Plot for the sequence. Use appropriate scale and spacing.
Practice.
Given the rule for a sequence \( f(n) = 18 + 2n \), create a table of values for the first six terms. Begin with \( n = 0 \).

<table>
<thead>
<tr>
<th>N</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

How would you describe this sequence/function?

Check with your neighbor. Do your descriptions agree? If necessary, make changes to your description to be precise.

For this function, what are the possible inputs?

Write one sentence beginning with “The domain of this function is …”.

For this function, what are the possible outputs?

Write one sentence beginning with “The range of this function is …”

How many different outputs will a position of \( n = 3 \) produce? Explain.

Construct a Scatter Plot for the sequence. Use appropriate scale and spacing.
1. Maria recorded her observations from her biology lab in the table shown.

<table>
<thead>
<tr>
<th>Time</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Use multiple representations to describe the sequence.

2. Write one sentence to describe whether or not this sequence is a function. Explain your response.

3. Write a sentence to describe the domain of this sequence.

4. Write a sentence to describe the range of this sequence.

5. Write two sentences explaining the information you would need to create a box plot for a set of data.

ACT Practice.

In the first year, the tuition at a local college is $4,000. If the tuition increases by $600 per year, how much will tuition be in the tenth year?
ACT Warm-Up
This table shows some terms of an arithmetic sequence. Fill in the missing terms.

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(n)</td>
<td>19</td>
<td></td>
<td>31</td>
<td></td>
<td>39</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is a function?

With your group, sort the information into two categories: functions and non-functions.

Sorting information.

Write additional notes or corrections based on class discussion.

Consider the following situation:
A blue pen costs $1.80. If we start with $10.00, how much money will we have left if we buy:

- Zero pens? \( f(0) \)
- One pen? \( f(1) \)
- Two pens? \( f(2) \)
- Three pens? \( f(3) \)
- Four pens? \( f(4) \)
- Five pens? \( f(5) \)

Use different representations to describe the sequence.

Explain whether or not the money left versus the amount of pens purchased is a function.

State the domain and range of this data.
Consider the following situation:
Marisa invests $300 at a bank that offers 5% compounded annually.
   How much money does Marisa have to start?
   How much money does Marisa have after 1 year?
   How much money does Marisa have after 2 years?
   After 3 years?
   After 4?
   After 10?

Use different representations to describe the sequence.

Answer the following questions based on the interest problem.

1. What is the range of this function?

2. What does \( f(0) \) mean? Is it on your scatter plot?

3. What does \( f(3) \) meant? Is it on your scatter plot?

4. Try to calculate \( f(20) \). Describe the meaning of \( f(20) \).
In 1803, President Thomas Jefferson negotiated what has become known as The Louisiana Purchase. In fact, the purchased involved much more than Louisiana, but Louisiana was a key issue because its transfer to U.S. ownership denied France and Spain the power to block American trade access to the port of New Orleans. In fact, the Louisiana Purchase doubled the size of the United States. The United States paid $15 million to France for land that included what now encompasses all or part of 15 states (AR, CO, IA, KS, LA, MN, MO, MT, ND, NE, NM, OK, SD, TX, WY) as well as land that eventually became part of two Canadian provinces (Albert and Saskatchewan).

Lately, France has had some financial problems. But suppose that, in January 1804, the French government had created a “rainy day” fund by depositing one-third of its income from the Louisiana Purchase in banks around the world. Also suppose that that investment had earned an interest rate of 3% compounded annually, and that none of that money was withdrawn before January 2011. What would that investment have been worth in January of 2011?
Notes from presentations.
1. Given the explicit definition for a sequence, \( f(n) = 4(1.5)^n \), create a data table for the first six terms. Use \( n = 0 \) as the starting position.

2. Write one sentence to describe the sequence in question 1.

3. Find \( f(8) \).

4. Find \( f(13) \).

5. Consider the following situation: A frozen pizza costs $5.05.
   What is the cost of zero pizzas?
   What is the cost of two pizzas?
   What is the cost of three pizzas?
   What is the cost of four pizzas?
   What is the cost of five pizzas?

6. Write one sentence to describe the cost of pizzas sequence.

7. Use function notation to write a recursive and an explicit definition for the sequence.

8. Find \( f(7) \), using your function from question 8.

9. Describe the domain and range for the situation in problem 5.

10. Write two sentences comparing the sequences in problem 1 and problem 5.

11. Explain whether or not the following is a function: The number of people at the Lagoon Park depends on the day of the week.

ACT practice.

Find the 100th term of this arithmetic sequence.

\[ 5, 8, 11, 14, 17, \ldots \]
ACT Warm-Up

Harold is starting a new workout program in which each day he will complete 4 more push-ups than the day before. If he starts with 5 push-ups on the 1st day, how many push-ups will he do on the 12th day?

Undefined geometric terms

Point
Line
Plane

Defined terms.
Segment
Circle

Arcs
Radius
Diameter

Congruent

Describe the following figures with precise language.
Introduction to Geometer’s Sketchpad.

1. Double-click on the icon for GSP with tiny shapes on it.
2. Click on the point tool in the left menu.
   a. Make 3 points on your screen.
   b. Name the points.
      i. Click on the A tool in the left menu.
      ii. Click once on each point.
      iii. If you want to change the name, double-click on the point and type in the new name.
3. Click on the line segment tool in the left menu.
   a. Click and drag on the screen to create a segment.
   b. Name the segment.
      i. Click on the A tool in the left menu.
      ii. Click once on each ENDPOINT.
   c. Measure the length of the segment.
      i. Click on the arrow tool in the left menu.
      ii. Highlight each ENDPOINT. The segment should not be pink.
      iii. In the top Measure menu, select distance.
4. Click on the line segment tool in the left menu.
   a. Hold down the button until you see different choices. Select the ray.
   b. Click and drag on the screen to create a ray.
   c. Name the ray.
      i. Click on the A tool in the left menu.
      ii. Click once on the endpoint and once on another point on the ray.
5. Click on the line segment tool in the left menu.
   a. Hold down the button until you see different choices. Select the line.
   b. Click and drag on the screen to create a line.
   c. Name the line.
      i. Click on the A tool in the left menu.
      ii. Click once two points on the line.
6. Click on the arrow tool in the left menu.
   a. Select the SEGMENT. Not the segment endpoints.
   b. In the construct menu at the top, select midpoint.
   c. The midpoint will be displayed on your segment.
   d. Name the midpoint M.
7. Click on the segment tool in the left menu.
   a. Hold down the button until you see different choices. Select the segment.
   b. Click and drag on the screen to make a polygon with 6 sides.
      i. Name the polygon vertices.
   c. Click and drag on the screen to make a polygon with 9 sides.
      i. Name the polygon vertices.
8. Click on the A tool in the left menu.
   b. Type your name and your class period.
   c. Double click near your 6-sided polygon: write the type of polygon.
   d. Double-click near your 9-sided polygon: write the type of polygon.
9. Click on the segment tool in the left menu.
   a. Click and drag to create an angle.
   b. Name the vertex and two other points on the angle (with A tool)
   c. Click on arrow tool in the left menu.
      i. In order, select a point on the side of angle, the vertex, then a point on the other side of the angle. Nothing else should be pink.
      ii. In the measure menu at the top, select angle.
10. Save to your personal school drive.
11. In the file menu at the top, select PRINT PREVIEW.
    a. Hit the FIT TO PAGE button at the top.
    b. Hit print.
**Interior angles of polygons Sketchpad exercise.**

Part 1: Finding the sum of the interior angles of a polygon.

1. Set preferences to allow for the automatic labeling of points and for angle measures in degrees.
   
   Tip: Under Edit… select Preferences…
   
   At the Text tab… make sure the “For all new points” is selected.
   At the Units tab… make sure the angles are in degrees.

2. Construct triangle ABC. Verify that the sum of the angles in a triangle equals 180 degrees by finding the size of each angle of the triangle and then finding the sum.
   
   Tip: to measure an angle, you must select three points with the vertex in the middle.

   Tip: to calculate the sum, go to measure… calculate. A calculator will appear. You may click on any measurement instead of typing in the number.

3. Construct quadrilateral ABCD.

   Tip: Create 4 points and connect in order with segments.

4. Choose a vertex of the quadrilateral and draw all diagonals from that point.

   Select A, B, C, or D and draw diagonals from that point to the other vertices.

5. Record the number of triangles that have been formed in the table.

6. Record the sum of the angles of all triangles in the table.

   Tip: multiply the number of triangles by 180.

7. Hide the diagonals you have drawn.

   Tip: select only those diagonals. In the display menu, select hide segments.

8. Find the total number of degrees of the interior angles of the polygon you are investigating.

   a. Measure each angle of the polygon (not the triangles).
   b. Sum up the measures of the angles.
   c. Record in the table.

9. Repeat steps 3 - 8 for each additional polygon listed in the table.
<table>
<thead>
<tr>
<th>Polygon</th>
<th>Sides</th>
<th>Triangles created by diagonal(s)</th>
<th>Sum of angles of all triangles (180 each)</th>
<th>Sum of the interior angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>1</td>
<td>180 degrees</td>
<td>180 degrees</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heptagon</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Octagon</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n-gon (what is your pattern?)</td>
<td>$N$ (any # of sides)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use different representations to describe the sequence found in the Sketchpad Exercise.

Explain whether or not the sum of the interior angles is a function of the number of sides of the polygon.

Find $f(32)$. What does that mean?

Find $f(101)$. What does that mean?
Notes from class discussion.
1. Write three sentences to describe what you found during the Sketchpad exercise.

What is the sum of the interior angles of a 15-gon?

2. Given the function \( f(n) = 19 + 2n \), create a data table with the first six terms.
   Use \( n = 0 \) as the starting position.
   Use different representations to describe the sequence in the data table.
   Does this data represent an arithmetic, geometric, or neither type of sequence?

3. Given the function \( f(n) = 3(1.7)^n \),
   Find \( f(1) \).
   Find \( f(4) \).
   Find \( f(2) \).
   Find \( f(w+1) \).

ACT Practice.

If \( h(x) = \frac{2x+3}{x+1} \), what is the value of the function \( h(y - 2) \) when \( y = 1 \)?

A. \( \frac{1}{2} \)  
B. \( \frac{3}{2} \)  
C. \( \frac{5}{2} \)  
D. Not defined
ACT Warm-Up

A career advisor tells Ming that a financial consultant earns $43,000 for the first year, and there is a 3% annual pay raise. If Ming takes a job as a financial consultant, what will be her highest annual salary after working a total of 35 years?

Write one sentence to describe the figure below.

The segment connecting points A and B divides the interior of the circle into two regions.

If a third point, C, were added (see below) and connected to both of the other labeled points (A and B), the resulting line segments would divide the interior of the circle into four sectors.

Suppose you continued adding a point for each of the other letters in the alphabet. As each new point is added, it would be connected by line segments to all the points already there. With your group, collect some data about the total number of regions created in the interior of the circle. Use multiple representations to describe the data.

Find f(26).

Is the data an arithmetic, geometric, or neither type of sequence?

Identify the domain and range of this function.
Notes from other presentations.
Consider the following two sets of data.

A) 

<table>
<thead>
<tr>
<th>Position</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

B) 

<table>
<thead>
<tr>
<th>Position</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
</tbody>
</table>

Write a sentence to describe sequence A.

Write a function to describe sequence A.

Write a sentence to describe sequence B.

Write a function to describe sequence B.

Write two sentences to compare the sequences.

Create a scatter plot of both sequences A and B on the same graph. Use appropriate scale and spacing. Also, be clear about which sequence is which on the scatter plot.

Write two sentences to describe the scatter plot.
Consider the following situation:

On July 11th, 2011, Kam and Kelly each had a unique experience. Kam received an anonymous check for $10,000 to be deposited in an account earning 2.5% interest, compounded annually. Kam was told he could not touch the money for 20 years. Kelly won an empty glass jar, with the guarantee that $18 would be placed in it, per week, for the next 20 years.

With your group, write a function for each “account” that describes the amount each has. Determine how much Kam and Kelly will each have after 20 years.

Which experience would you rather have, that of Kam or that of Kelly? Explain. Which account grows faster?
Write two sentences to explain whether or not you have changed your preferred plan (Kam or Kelly’s).
Practice Set 14

1. Given the sequence below,

   Write one sentence to describe the data.

   Write a recursive formula to describe the data.

   Write an explicit definition to describe the data. Find \( f(11) \).

<table>
<thead>
<tr>
<th>Position</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>4</td>
<td>12</td>
<td>36</td>
<td>108</td>
<td>324</td>
</tr>
</tbody>
</table>

2. Given the function \( g(n) = \frac{n + 7}{n - 2} \), find \( g(4) \).

3. Given the following data set, find the median, min, max, and interquartile range. Create a box plot.

   The average waiting time for a city bus from random sample on 12 different days. The data are: 10 minutes, 6 minutes, 8 minutes, 23 minutes, 6 minutes, 14 minutes, 17 minutes, 10 minutes, 26 minutes, 20 minutes, 5 minutes, 5 minutes.

4. What’s the better deal, earning $1000 per day for the rest of your life or earning $.01 the first day, and doubling it every day for the rest of your life? How do you know? Do you think an 80-year-old woman would make the same choice? Should she?

ACT Practice.

Given an arithmetic sequence in which \( a_1 = 3, a_5 = 19, \) and \( a_n = 163 \), what is \( n \)?
ACT Warm-Up
How many terms are in the arithmetic sequence 3, 10, 17, …, 73.

Use the following graph to answer the questions below.

1. What is the price of a 3-ring binder?
2. What is the total price of 4 binders?
3. How many binders could we buy for $11.25
4. How many binders could we buy for $10.00
5. Could a straight line be drawn exactly through all of the points that are plotted on the graph?
6. Why is the line not shown?
7. What is the domain of the function?
8. Finish the statement: As the number of binders increases by one, the total price
9. Write a function for the data.
10. What would be a situation where we would draw a line through the data?
The cost of bananas is $1.20 per pound.
  1. What is the cost of one pound of banana?
  2. What is the cost of three pounds of bananas?
  3. What is the cost of five pounds of bananas?
  4. What is the cost of zero pounds of bananas?

Use different representations to explain the data. Include a scatter plot of the sequence.

5. How many pounds of bananas could we buy for $2.40?

6. How many pounds of bananas could we buy for $7.20?

7. Is it possible that a straight line could be drawn exactly through all of the points that are plotted on the graph?

8. Would it be appropriate to draw a line? Explain.

Make additions or corrections to your explanation based on class discussion.

9. What is the domain of the function?

10. Finish the statement: *As the number of pounds of bananas increases by one, the total price* 

11. Write two sentences to explain how the scatter plots (and functions) compare for the 3-ring binder problem and the cost of bananas problem.
Bouncing Ball Investigation

1. Use different representations to describe the collected data.

2. Could a straight line be drawn exactly through all of the points that are plotted on the graph?

3. Could a smooth curve be drawn exactly through all of the points that are plotted on the graph?

4. Would it be appropriate to draw a smooth curve? Explain.

5. Make additions or corrections to your explanation based on class discussion.

6. What is the domain of the function? What is the range of the function?

7. Finish the statement: *With each bounce, the height of the ball...*
Notes from class discussion of lab.
1. On your way home, note the cost of one gallon of diesel fuel. _________
   What is the cost of 0 gallons of diesel fuel?
   What is the cost of 8 gallons of diesel fuel?
   What is the cost of 12 gallons of diesel fuel?
   What is the cost of 16 gallons of diesel fuel?
   What is the cost of 20 gallons of diesel fuel?

2. Create a scatter plot of the sequence. Use appropriate scale and spacing.

3. How many gallons of fuel could we buy for $10.00?

4. How many gallons of fuel could we buy for $20.00?

5. Is it possible that a straight line could be drawn exactly through all of the points that are plotted on the graph?

6. Is the data discrete or continuous? Would it be appropriate to draw a line? Explain.

7. What is the domain of the function?

8. Finish the statement: As the number of gallons of diesel fuel increases by one, the total price

9. Write a function for the data.

10. Given a recursive formula for a sequence, create a data table with the first six terms, beginning with $n = 0$. $f(n) = f(n-1) + 12$, with $f(0) = -4$.  
    (cont.)
11. Write two sentences to compare the data shown in the scatter plots below.

12. For the given data set, find the median, min, max, and interquartile range. Draw a box plot.

The marks of seven students in a mathematics test with a maximum possible mark of 20 are given below:
15 13 18 16 14 17 12

13. Write two sentences to describe the figure shown below. Use precise language.

ACT Practice.

The cost of a single-scoop ice-cream cone is $2.00. Each extra scoop of ice cream costs an additional $1.50. If $x$ is the number of extra scoops of ice cream, and $y$ is the total cost of an ice-cream cone, which graph most accurately represents this situation?
ACT Warm-Up

A total of $9,100 in prizes is awarded to the top 8 entries in a science project contest. The highest prize is $2,000. The differences between successive prizes are equal. What is the amount, in dollars, of the lowest prize?

As a group, refer to data for the number of cell phone users from 1986 to 1994.

<table>
<thead>
<tr>
<th>Years after 1986</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Cell Phone Users</td>
<td>498</td>
<td>872</td>
<td>1527</td>
<td>2672</td>
<td>4677</td>
<td>8186</td>
<td>14325</td>
<td>25069</td>
<td>43871</td>
</tr>
</tbody>
</table>

1. Use different representations to describe the data.

2. Could a straight line be drawn exactly through all of the points that are plotted on the graph?

3. Could a smooth curve be drawn exactly through all of the points that are plotted on the graph?

4. Would it be appropriate to draw a smooth curve? Explain using precise vocabulary.

5. What is the domain of the function? What is the range of the function?

6. Finish the statement: *As the number of years increases by one, the number of cell phone users...*

7. Find f(25). Explain the meaning of f(25).
Joan noticed that it took eight years for her $2500 stock investment to double. The function \( M(t) = 2500 \cdot 2^t \) could be used to model the growth of her investment if it continues to grow at the same rate. Use the function to determine the value of her investment after:

1. \( t = 0 \) years
2. \( t = 8 \) years
3. \( t = 10 \) years
4. \( t = 12.5 \) years
5. \( t = 18.7 \) years
6. Use different representations to describe the data.

7. Could a straight line be drawn exactly through all of the points that are plotted on the graph?
8. Could a smooth curve be drawn exactly through all of the points that are plotted on the graph?
9. Would it be appropriate to draw a smooth curve? Explain.
10. Make additions or corrections to your explanation based on class discussion.
11. What is the domain of the function? What is the range of the function?
12. Write two sentences to explain how the scatter plots (and functions) compare for the cell phone problem and Joan’s investment problem.
Write two sentences to describe the difference between discrete and continuous data.

Compare your response with your neighbor. Make any additions or corrections to your response.

Write additional comments from class discussion.

For each scenario, write one sentence to explain whether you believe the data collected is discrete or continuous.
1. Louis graduated college and bought himself a car. He owes $15,700 to the bank, but makes a $400 payment each month. A function to track the balance of the loan is \( f(m) = 15,700 - 400m \) where \( m \) is the number of months.

2. Cherie drinks a caffeinated Dr. Pepper before she starts working on her term paper entitled *Literary Giants in the Great Gatsby*. As time passes, the caffeine in her bloodstream decreases. After an hour, 13% of the caffeine is gone. A function to monitor the amount of caffeine in the bloodstream is \( f(h) = 130(0.87)^h \).

Create one scenario which would produce discrete data. Write two sentences to describe the scenario.

Create one scenario which would produce continuous data. Write two sentences to describe the scenario.

Activity: One student reads their scenario and the class votes by moving to one side of the room or the other (Discrete or Continuous).

Write comments from discussion on how discrete or continuous data affects domain and range.

*Introduce Illuminations: Drug Filtering*
Write three sentences to summarize the findings of the Drug Filtering exercise.

Write a function that models the sequence of data found in the Drug Filtering exercise.

Use your function to find \( f(72) \). Explain the meaning of \( f(72) \).

Given the following functions for data collected in various exercises, write two sentences explaining a starting amount and a change.

1. \( f(t) = 33 + 2t \), where \( f \) is the amount of fluoride (in micrograms) and \( t \) is the number of toothpaste applications.

2. \( A(m) = 900 - 150m \), where \( A \) is the amount in a debit account and \( m \) is the number of months.

3. \( R(t) = 200(.5)^t \), where \( R \) is the amount of radioactive material and \( t \) is the time in years.

4. \( B(h) = 4(3)^h \), where \( B \) is the number of bacteria and \( h \) is the number of hours.

Make any additions or corrections to your responses after class discussion.
For each of the following, write a sentence explaining why you think the data is either discrete or continuous. Write one sentence that describes the domain.

1.

2.

3. Jen’s soccer team has a fundraiser selling cookie dough to pay off her sports fee of $250. For every bucket of dough that she sells, her fee is reduced by $7.50.

Refer to your work with the Drug Filtering activity to answer the following questions.

4. Was the amount of medication increasing or decreasing with time?

5. Would the rate of change be different if a larger amount of a drug were initially administered? Explain.

Given a function, \( B(h) = 1350 - 22h \).

6. Write two sentences to describe a scenario that this function could represent.

7. Find \( B(10) \). Explain the meaning of \( B(10) \) in terms of your scenario.
What does **linear** mean?

Write one sentence to describe any thoughts you have on which (if any) sequences, data sets, or functions we’ve studied, have been linear.

Compare with a neighbor. Make additional comments or corrections to your response.

Additional comments from class discussion.

Determine whether the following are linear or non-linear.

1. Julia has 13 foreign coins in her collection. On average, she adds 2 new coins per year.

2. \( f(n) = 11(2)^n \)

3. Boris’ deposit $500 in an account. The bank offers a 4.2% interest rate compounded annually.

4. [Graph]

5. | Position | 0 | 1 | 2 | 3 | 4 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>95</td>
<td>91</td>
<td>87</td>
<td>83</td>
<td>79</td>
</tr>
</tbody>
</table>

Add any helpful notes.
The Fillmans bought a new hot tub and filled it with water from the garden hose. Initially, the temperature of the water was 56 degrees. When they turned on the tub, the temperature rose about 4 degrees every hour.

What is the starting amount?

We call this the intercept.

Is this amount increasing or decreasing?

How many degrees does the temperature change every hour?

This constant change is called the slope.

Is this function linear? Explain.

Write a function to describe the temperature of the tub.

Find six pieces of data from this function.

Draw a scatter plot. Use appropriate scale and spacing.

Is this data discrete or continuous? Explain.

Does your scatter plot reflect your response whether this function is linear or not?

Where does the intercept appear on your scatter plot?

Do you think an intercept will always be in that position? Explain.

What is f(5)? Explain the meaning of f(5).
As a group, consider the following situation:
Andy wants to drop weight for wrestling. He weighs 187 now, and plans to lose 1.5 lbs. per week.

What is the intercept?

Is this amount increasing or decreasing?

How many pounds does the weight change every week?

Write a sentence beginning with “The slope of this function is …”.

Is this function linear? Explain.

Write a function to describe the Andy’s weight as time passes.

Find six pieces of data from this function.

Draw a scatter plot. Use appropriate scale and spacing.

Is the data discrete or continuous? Explain.

Explain the domain and range for this scenario.

Where does the intercept appear on your scatter plot? Does this agree with your thoughts from the last problem?

What is f(5)? Explain the meaning of f(5).
Notes from class discussion.
Practice Set 17
Name ________________________________

1. What is a linear function?

2. Create a scenario that could be represented with a linear function. Use at least two sentences to describe it.

3. Write a function to describe the scenario you created in problem 2.

4. Find f(9). Explain the meaning of f(9).

5. Given a function, \( H(p) = 1.5 + .75p \), where H is the number of hours of work and p is the number of people participating in the race, write a sentence that describes the intercept and the slope.

6. Consider the sequence given below.

<table>
<thead>
<tr>
<th>Position</th>
<th>0</th>
<th>1</th>
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<th>3</th>
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<tr>
<td>Term</td>
<td>701</td>
<td>801</td>
<td>901</td>
<td>1001</td>
<td>1101</td>
</tr>
</tbody>
</table>

Write a sentence to describe the sequence.

Write a recursive formula to describe the sequence.

7. Given the following two histograms, write two sentences to compare them.

ACT Practice.
Find the product of the mean and the median of the data set.
31.8 39 32 30.7 37 36.3 29.9
Recall how you know if a function, sequence, scatterplot is linear or non-linear. Write two sentences.

Check with a neighbor to make additional notes or corrections.

Consider the following sequences. With your neighbor, work through the first several prompts.

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<thead>
<tr>
<th>Position</th>
<th>0</th>
<th>1</th>
<th>2</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>2</td>
<td>3.2</td>
<td>4.4</td>
<td>5.6</td>
<td>6.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>2</td>
<td>2.4</td>
<td>2.88</td>
<td>3.456</td>
<td>4.1472</td>
</tr>
</tbody>
</table>

Write one sentence to describe the sequence. Write one sentence to describe the sequence.

Is this sequence linear or non-linear? Explain. Is this sequence linear or non-linear? Explain.

Write a function to describe the sequence. Write a function to describe the sequence.

Write two sentences to compare the two sequences.

Which sequence do you think will reach 100 first? Explain.
Look at scatter plots to determine the correct response.

Make additional notes or corrections during class discussion.

Linear functions and Exponential functions

Write a sentence to describe the difference between the slope or change of a linear function and the change of an exponential function.

Write a sentence to compare the intercept or starting point of a linear function and an exponential function.

Determine whether or not the following are exponential functions.

1. Julia has 13 foreign coins in her collection. On average, she adds 2 new coins per year.

2. \( f(n) = 11(2)^n \)

3. Boris’ deposit $500 in an account. The bank offers a 4.2% interest rate compounded annually.
As a class, determine if the data could be described with a function.

Is this data an exponential function? Explain.

As a group, consider the following situation.
A certain strain of bacteria that is growing on your kitchen counter doubles every minute. Assuming that you start with only one bacterium, how many bacteria could be present at the end of 8 hours?

Is this situation an exponential function? Explain.

What is the intercept of this function?

Is the amount of bacteria increasing or decreasing? How do you know?

What is the rate of change of this function?

Write a function to describe the bacteria count on your kitchen surface.

Find f(8 hours).
As a class, use the scatter plot applet to make a quick sketch of the function. Should the function be described as discrete or continuous? Explain.

As a group, consider the situation. Matt bought a new car at a cost of $25,000. The car depreciates approximately 15% of its value each year.

Is this situation an exponential function? Explain.

What is the intercept of this function?

Is the amount of bacteria increasing or decreasing? How do you know?

What is the rate of change of this function?

Write a function to describe the value of Matt’s car as time passes.

Find \( f(8 \text{ years}) \).

As a class, use the scatter plot applet to make a quick sketch of the function. Should the function be described as discrete or continuous? Explain.
Write one sentence to describe how you will tell if an exponential function is increasing or decreasing.

Make additional comments or corrections based on class discussion.

Write an exponential function that is increasing.

Write an exponential function that is decreasing.

Write a linear function that is increasing.

Write a linear function that is decreasing.

Write two sentences to explain your understanding of linear and exponential functions, their similarities and differences.
1. Write two sentences to compare linear and exponential functions.

For each of the following, decide whether a linear function, exponential function, or something else all together is depicted. Explain your response.

2. Tino drinks strawberry shakes to increase muscle mass. Advertisements for the shakes say that for every serving, muscle mass increases by 2%.

3. Michelle babysits on Thursday afternoons to save money for the summer biology trip. She currently has $114 and makes an additional $25 each Thursday.

4. 

5. \( f(t) = 13(1.07)^t \)

6. Given a function, \( f(n) = 13 + 4n \),
   Write a sentence to describe if it is linear, exponential, or something else.
   Write a sentence to describe the intercept and slope or change.

   Find \( f(1.3) \).

7. Given a function, \( W(r) = 17(.67)^r \),
   Write a sentence to describe if it is linear, exponential, or something else.
   Write a sentence to describe the intercept and slope or change.

ACT Practice.

Jenny invested $300 in an account where her money doubles every 7 years. How much money will be in her account at the end of 35 years?

A. $ 4,200
B. $ 9,600
C. $21,000
D. $38,400
ACT Warm-Up

Consult with your group to write a comparison of linear and exponential functions. Include the vocabulary: intercept, slope or rate of change. Also include a description of the scatter plots for each type of function.

Make additional comments or corrections during class discussion.

_ILLUMINATIONS: Devil and Daniel Webster activity_

Write the functions that you developed for the Devil and Daniel Webster activity.

Write two sentences to compare the two functions.

What type of functions did you write? Explain.

Define the domain and range for each function.
Consider the following two situations.
A) Gert starts with $42 on her lunch balance, and is charged $2.50 everyday she eats at the cafeteria.

B) 

<table>
<thead>
<tr>
<th>Days</th>
<th>Account</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>42</td>
</tr>
<tr>
<td>1</td>
<td>44.50</td>
</tr>
<tr>
<td>2</td>
<td>47.00</td>
</tr>
<tr>
<td>3</td>
<td>49.50</td>
</tr>
</tbody>
</table>

Write a sentence to describe the first situation.

Write a sentence to describe the second situation.

Write two sentences to compare the two situations.

Make additional comments or corrections during class discussion.

Work with your group to determine which is a better plan to decrease the rat population in the basement of town hall. The rat count is currently at 311.
A) Trap and remove 2 rats every day.
B) Use a chemical that removes 1% of the population each day.
1. Write a comparison of linear and exponential functions. Include the vocabulary: intercept, slope or rate of change. Also include a description of the scatter plots for each type of function.

2. Consider the following two situations.
   A) Bella has a small collection of thimbles, but would like to build a bigger collection. She currently has 12, but figures she can purchase 1 each week, thanks in part to her new job delivering newspapers.
   B) Teddy Bears

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Teddy Bears</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

Write a sentence to describe the first situation.

Write a sentence to describe the second situation.

Write two sentences to compare the two situations.

3. Determine the intercept and rate of change for the function \( f(n) = 15(1.04)^n \).

4. Find the median and IQR for the following data set.
   65.4  62.7  61.1  68.3  62.5  62.5

ACT Practice.

An artist slices a cone along a plane that is not parallel to the base of the cone, as shown in the figure. What cross section of the cone does the artist create?
ACT Warm-Up

Jaci is given the following true statements:

1. The sophomore class will win a pizza party if and only if they win the school magazine subscription sales challenge.

2. The sophomore class will win the sales challenge if and only if they sell the most magazine subscriptions.

3. The junior class sold more magazine subscriptions than the sophomore class.

What can Jaci conclude?

A. The junior class will win the pizza party.
B. The sophomore class will win the pizza party.
C. The junior class will not win the pizza party.
D. The sophomore class will not win the pizza party.

List coordinates of vertices of triangle.

Find length of each side.

Find the area of the triangle.

Move the entire triangle right three spaces.
List coordinates of vertices.

How are the coordinates different from the original vertices?

Notation: \( f(x, y) \rightarrow (x + 3, y) \)

Will this action ever take the point \((0, 0)\) and move it to something other than \((3, 0)\)? Is this action a function?
If this is a function, what is the domain?

Length of each side.

Area of triangle.

Write a sentence to describe whether the translated triangle is the same or different in size and area as the original.

**NLVM translation**

Consider a rectangle ABCD whose vertices are located at (0,0), (4,0), (4,2), (0,2) respectively.
Graph the rectangle.
Find length of each side.

Find the area of the rectangle.

Move the entire rectangle up three spaces on the graph.

List coordinates of vertices.

How are the coordinates different from the original vertices?

Notation: \( f(x, y) \rightarrow (x, y + 3) \)

Will this action ever take the point (0,0) and move it to something other than (0,3)? Is this action a function?

If this is a function, what is the domain?

Length of each side.

Area of rectangle.

Write a sentence to compare the translated rectangle to the original.
As a group, consider a rectangle LMNP with vertices L(1,2), M(6,2), N(6,8), P(1,8).
Graph the rectangle and verify its properties.

Find length of each side.

Find the area of the rectangle.

Move the rectangle to the left two spaces and up three spaces.

List coordinates of vertices.

How are the coordinates different from the original vertices?

Use notation: \( f(x,y) \rightarrow \)

Will this action ever take the point (1,2) and move it to something other than (-1,5)? Is this action a function?

What is the domain?

Check slopes for perpendicular lines.

Length of each side.

Area of rectangle.

Write a sentence to compare the translated rectangle to the original.
Practice Set 20  Name ________________________

1. Write one sentence to describe how you will know if a collection of data represents a function or not.

2. Fill in the table with data that represents a function.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Fill in the table with data that does NOT represent a function.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Write one sentence to describe how the original figure was translated to the new figure.

5. Use function notation to describe the translation.

6. Write one sentence to compare the two figures. (You do not need to perform calculations for this comparison).

7. Determine the intercept and rate of change for the function \( f(n) = 22.7 + 1.4n \).

8. Determine the intercept and rate of change for the function \( f(n) = 10,000(.85)^n \).

ACT Practice.

An internet provider sells internet packages based on low monthly rates. The price for the internet service depends on the speed of the internet connection. The chart below indicates the prices of the various internet packages.

<table>
<thead>
<tr>
<th>Gigabyte speed</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price in dollars</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
</tbody>
</table>

Write a function to represent the prices of these internet packages.  

133
When a student subtracts 18 from a number, the result is \( \frac{1}{4} \) of the number. What is the number?

A. 6  
B. 18  
C. 24  
D. 36

Consider a parallelogram ABCD with vertices… A(0,0), B(7,0), C(10, 4), D(3,4).

Graph the parallelogram.

With your group, discuss why this figure is called a parallelogram. Write your ideas below.

Move the parallelogram two spaces to the right and four spaces up.

List coordinates of vertices.

How are the coordinates different from the original vertices?

Use notation: \( f(x, y) \rightarrow \)

Will this action ever take the point (10, 4) and move it to something other than ( )?

Is this action a function?

What is the domain of this function?

Are the two figures congruent?
Given a domain of a circle whose center is at (2,3) and radius is 4, Graph the circle.

On the same graph, illustrate the following function. 
\[ f(x, y) \rightarrow (x - 3, y - 2) \]

What is the range of this function?

Are the two circles congruent? How do you know?

Given a table of values, define the function with proper notation. 

<table>
<thead>
<tr>
<th>Domain</th>
<th>(1,7)</th>
<th>(3,5)</th>
<th>(4,2)</th>
<th>(-3, 8)</th>
<th>(0,11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>(3,4)</td>
<td>(5,2)</td>
<td>(6,5)</td>
<td>(-1,5)</td>
<td>(2,8)</td>
</tr>
</tbody>
</table>

worksheet practice.

The original design for the new community swimming pool placed the structure very close to the home of Mr. Ralph Maggia. After several town meetings, Mr. Maggia convinced the community board that the pool design was terrific, but needed to be moved further from his home. Specifically, he requested that it be moved at least 120 feet to the east and 90 feet north. Write a function to describe the minimum move of the swimming pool.
1. Given the figure, translate the circle according to the following function. 
\[ f(x, y) \to (x + 6, y). \]

2. Calculate the radius of each circle. Use this information to write a sentence to describe the two circles as congruent or not congruent.

3. Given a table of values, determine whether or not it represents a function. Explain.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,7)</td>
<td>(3,4)</td>
</tr>
<tr>
<td>(1,7)</td>
<td>(3,8)</td>
</tr>
<tr>
<td>(4,-2)</td>
<td>(6,-5)</td>
</tr>
<tr>
<td>(-3, 8)</td>
<td>(-1,5)</td>
</tr>
<tr>
<td>(0,11)</td>
<td>(2,8)</td>
</tr>
</tbody>
</table>

4. Given a table of values, determine whether or not a function can be written. If it is possible, write the function.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>(-2,4)</td>
</tr>
<tr>
<td>(3,2)</td>
<td>(1,6)</td>
</tr>
<tr>
<td>(4,-8)</td>
<td>(2,-4)</td>
</tr>
<tr>
<td>(-2, 5)</td>
<td>(-4,9)</td>
</tr>
<tr>
<td>(9,1)</td>
<td>(7,5)</td>
</tr>
</tbody>
</table>

5. Given the function, \( f(n) = 18 - n \), write one sentence to describe it. Include information about intercept and rate of change or slope.

ACT Practice.

Raul is training for a marathon. One day he runs 4 miles north, then 2 miles east, then 8 miles north, and 3 miles east. He returns to his starting point by running along the shortest possible direct path. To the nearest mile, what is the total distance of Raul's run?
ACT Warm-Up

What is the decimal equivalent of $\frac{1}{12}$?

A. .083  B. .0083  
C. .12   D. .012  
E. None of the above

Vectors can describe translations

Notation

Magnitude

Examples.
A point has been translated from the origin to the point (4, -11). Write the translation as a vector. Find the magnitude of the vector to calculate the distance.

Find $|\mathbf{v}|$ if $\mathbf{v} = \langle 7, 12 \rangle$.

Write the components of a vector whose initial point is (-7,2) and whose terminal point is (5,-3).
As a group, create pairs of initial and terminal points that represent the vector \( \mathbf{v} = <-2,5> \).

Notes from class discussion.
Consider the following situation.
A triangle with vertices A(5, 1), B(1, 4), and C(2, 8) is translated according to the following function \( f(x, y) \rightarrow (x + 2, y + 4) \). The resulting image is then translated according to the following vector \( \mathbf{u} = <-2, -4> \). Use different representations to describe the translations.

Notes from class discussion.

Introduction to vector operations
1. Write one sentence to describe how a vector can be used to describe a translation.

2. A point has been translated from (-9, -4) to the point (3, 1). Write the translation as a vector. Find the magnitude of the vector.

3. Draw vectors to represent the following situation. Jean walks from her home 2 miles east. She then turns around and walks 3 miles west to her friend Sam’s house.

   What is the total distance that Jean walked?

   What is the displacement or distance from where she started to where she ended up?

4. Write two sentences to compare linear and exponential functions.

ACT Practice
What is the percent equivalent of 0.2184?

A. 0.002184%
B. 0.02184%
C. 0.2184%
D. 2.184%
E. 21.84%
ACT Warm-Up
If \( 3x - 2 + 4x = 5 \), then \( x = ? \)

A. 1    B. \(-1\)
C. 7    D. 3
E. \(-3\)

What is a vector?

Is it possible to combine multiple translations/vectors using addition?

**Headwind, tailwind, crosswind**

As a group, use multiple representations to describe example 1 of the webpage.
Example. A car is travelling north at 45 mph and collides into another car travelling east at 30 mph. Represent the collision graphically.

Example. Given the vectors \langle 4,7 \rangle and \langle -1,2 \rangle, select a method to find their sum. What is the magnitude of the sum? Is this magnitude the same as the sum of the individual vectors’ magnitudes?
Consider the point Q(5,3). Apply the vector \( \mathbf{u} = \langle 2, -1 \rangle \). Use different representations to display the transformation.

Find \( |\mathbf{u}| \).

Consider the point Q(5, 3). Apply double the vector 2\( \mathbf{u} \). Use different representations to display the transformation.

Find the magnitude of 2\( \mathbf{u} \).

Consider the point Q(5, 3). Apply the opposite vector, -\( \mathbf{u} \). Use different representations to display the transformation.

Find the magnitude of -\( \mathbf{u} \).
Write one sentence to describe the effects of scalar multiplication of a vector.

Write one sentence to compare scalar multiplication of a vector to that of a matrix.
1. Use different representations to show the operations listed, given \( \mathbf{u} = <3, -1> \) and \( \mathbf{v} = <6, 2> \).

   a. \( |\mathbf{u}| \) and \( |\mathbf{v}| \)

   b. \( \mathbf{u} + \mathbf{v} \)

   Find the magnitude of the sum. Write one sentence to compare this sum to the responses in part a.

   c. \( 2\mathbf{v} \)

   d. \( -4\mathbf{u} \)

   e. Write one sentence to describe how to change the direction of a vector, but not the magnitude.

2. A soccer player runs forward a distance of 4 m, reverses direction and runs a distance of 3 m, and then reverses direction again and runs a distance of 8 m.

   a. What distance does the player run?
   b. What is his displacement?
3. A hiker starts by walking along a straight path. He then turns and walks 260.0 m west. If he finds he is located 360.0 m exactly north of his starting point, what was his displacement along the path?

4. Use precise language to describe the following figure.

![Figure](image)

ACT Practice If $7x + 6 = 27$, the $x = ?$

A. 2
B. 3
C. 9
D. 13
E. 21
ACT Warm-Up

These stem-and-leaf plots show the number of cloudy days and the number of partly cloudy days for each month of a recent year.

<table>
<thead>
<tr>
<th>cloudy</th>
<th>partly cloudy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 9</td>
<td>0 6 6 8 8 9 8</td>
</tr>
<tr>
<td>1 0 0 1 2 2 3 5 6 6 8</td>
<td>1 3 4 5</td>
</tr>
</tbody>
</table>

Which statement is true based on the information in the stem-and-leaf plots?

A. The total number of cloudy days equals the total number of partly cloudy days.
B. The mean number of cloudy days is greater than the mean number of partly cloudy days.
C. Every month of the year has at least 10 cloudy days.
D. The mode number of cloudy days equals the mode number of partly cloudy days.

Write one sentence using the vocabulary: translation, function, congruent.

Write a series of functions to track the movement of the penguin from positions A through F.

Compare your functions with a neighbor’s functions. If possible, make corrections to improve your function path.

Consider the following figure. Determine whether it is possible to translate the original octagon to the new octagon. Write one sentence to explain.
Reflections demo and sketchpad exercise.

Write two sentences to describe what reflections do.

Write one sentence to say whether or not reflections can be modeled with functions.

Make additional comments or corrections during class discussion.

**NLVM reflection** to demonstrate again

Write one sentence to answer the following question: Do you believe that reflections produce congruent figures?

Write one sentence to describe a method to verify whether the figures are congruent or not.

As a class, decide on a method to verify congruence. Write one sentence to summarize the method.

Practice.
Triangle HVM has vertices at (-1,7), (2,4), and (4,5). Reflect triangle HVM across the x-axis.

Write a function that describes the reflection.
Use the class techniques to verify that the triangles are congruent.

Triangle ABC has vertices at (1,3), (4,7), and (5,0). Reflect triangle ABC across the y-axis.

Write a function that describes the reflection.

Use the class techniques to verify that the triangles are congruent.

Use one sentence to summarize whether or not reflections result in congruent figures.
Practice Set 24  Name _______________________________

1. Write two sentences to describe your class method of verifying congruent figures.

2. Draw a square figure on a coordinate system. Write the coordinates of the vertices.

   Draw a new figure following the function \( f(x, y) = (x + 2, y + 1) \) on the same coordinate system. Shade this figure.

   Write the coordinates of the vertices of the new figure. Do these coordinates follow the function?

   Write one sentence to describe the effect of this function.

   Draw a new figure that uses the shaded figure as the domain of the function \( f(x, y) \rightarrow (x, -y) \) on the same coordinate system.

   Write one sentence to describe the effect of this function.

3. Given the sequence, write an explicit formula to describe it.

<table>
<thead>
<tr>
<th>Position</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>22</td>
<td>11</td>
<td>5.5</td>
<td>2.75</td>
<td>1.375</td>
</tr>
</tbody>
</table>

4. Write two sentences to describe the scatter plot of the data modeled by this function: \( f(n) = 45(.99)^n \).

ACT Practice  The length of a rectangle is 10 cm, and the width is 14 cm. The length of a diagonal is given by \( \sqrt{14^2 + 10^2} \). To the nearest tenth of a centimeter, what is the length of a diagonal?
ACT Warm-Up

On the \((x,y)\) coordinate plane, \(\triangle ABC\) has vertices at \(A(-6,5), B(-3,-2),\) and \(C(1,2)\). What is the length of the segment that joins vertex \(A\) with the midpoint of \(BC\) ?

Given the following quadrilateral and its reflection, discuss methods to find the mirror line. Take notes on the discussion.

Make additional comments or corrections during discussion.
For each figure, describe where a mirror line would have to be to reflect the image onto itself?

(Images on Sketchpad)

Lines of symmetry

Write one sentence to summarize how these lines of symmetry were found.
Given the data set, decide whether it describes a reflection function or a translation function.

Translation powerpoint.

Write one sentence to help someone identifying a reflection function.

Write one sentence to help someone identifying a translation function.

Write a function to describe each transformation.

1. A figure on the coordinate system shifts 16 units to the right and 8.5 units down.

2. The points A(1,7), B(2, 4), and C(3,2) are reflected across the y-axis.

3. The figure is reflected about the dotted line.

4. The figure is transformed through the process described in the diagram.
1. Write two sentences to describe why midpoints are helpful in finding the mirror line of a reflection.

2. Write a function that maps the point (7,2) to the point (10,-3).

3. Find the mirror line for the image and its reflection.

5. Create a data table with the first six terms of the sequence defined by the following recursive formula.
   \[ f(n) = 2 \cdot f(n-1) + 1 \] where \( f(0) = 3 \).

6. Given the following data set, find the median, min, max, and interquartile range.
   Create a box plot.
   \[
   \begin{array}{cccccc}
   103 & 112 & 105 & 108 & 99 & 81 \\
   \end{array}
   \]

ACT Practice.

The slope of \( \overline{CD} \) is \( \frac{4}{5} \) and the slope of \( \overline{AB} \) is \( -\frac{5}{4} \).

What can be concluded about \( \overline{CD} \) and \( \overline{AB} \)?
Write two sentences to describe what rotations do.

Write one sentence to say whether or not rotations can be modeled with functions.

Make additional comments or corrections during class discussion.

Write one sentence to answer the following question: Do you believe that rotations produce congruent figures?

Triangle ABC has vertices at (1,3), (4,7), and (5,0). Rotate triangle ABC 180° about the origin.

Write a function that describes the rotation.
Use the class techniques to determine whether or not the triangles are congruent.

A square has vertices at (1,0), (4,0), (4,3), and (1,3). Rotate the square 90° about the origin.

Write a function that describes the rotation.

Use the class techniques to determine if the squares are congruent or not.
1. Create a quick sketch of angles that are approximately the given measure.
   \[ 90^\circ \quad 45^\circ \quad 10^\circ \]

2. Write a function that describes a point’s \(90^\circ\) rotation about the origin.

3. A triangle has vertices at \(A(2, 1), B(5, 1),\) and \(C(4,2)\). Graph the triangle. Then rotate the triangle \(180^\circ\) about the origin.

4. Write a function that describes a translation of a triangle 3 units to the right and 4 units down.

5. Create a data table with the first six terms of the sequence defined by the following recursive formula.
   \[
   f(n) = \left(\frac{1}{2}\right) \cdot f(n-1) \quad \text{where} \quad f(0) = 80.
   \]

6. Given the following data set, calculate the mean and standard deviation.
   \[
   103 \quad 112 \quad 105 \quad 108 \quad 99 \quad 81
   \]

ACT Practice.

The vertices of \(\triangle ABC\) are \(A(-4,2),\) \(B(-1,-3),\) and \(C(6,5)\). The triangle is translated 3 units to the left and then reflected about the line \(y = 1\). What is the final location of point \(B\)?

A. \((-4,-5)\)
B. \((-4,-2)\)
C. \((-4,3)\)
D. \((-4,5)\)
What are the coordinates of the image of \( \triangle XYZ \) after a reflection across the y-axis followed by a rotation of 90° clockwise about the origin? Show your work, including a graph of each transformation, and explain how you found your answer.

As a group, center each figure on its own coordinate system. If possible, determine an amount of rotation about the origin that would rotate the figure onto itself.
Notes from class discussion.

Write several sentences to compare translations, reflections, and rotations.
1. Rotate the following figure 180° about the origin.

2. Reflect the following figure about the x-axis.

3. Perform the following function on the given figure. \( f(x, y) \rightarrow (x + 4, y + 2) \).

4. Write one sentence to define a function. Using the data tables, create an example of a function and a non-function.

<table>
<thead>
<tr>
<th>Step</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(over for additional practice)
5. Given a function defined by $f(n) = 13 + 7n$, create a data table of values. Begin with $n = 0$.

<table>
<thead>
<tr>
<th>Step</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

6. Determine whether each of the following is a linear function, exponential function, or neither. Explain your responses.
   a. $f(n) = f(n-1) + 5$, with $f(1) = 4$.
   b. $f(n) = x^2 + 5$

   c. | Step | f(n) |
      |------|------|
      | 0    | 3    |
      | 1    | 6    |
      | 2    | 12   |
      | 3    | 24   |

7. Write one sentence to describe any differences between symmetric and skewed data.

ACT Practice

This graph shows the heat index versus relative humidity at a temperature of 80°F.

What is the slope of the line containing point T and point U?
ACT Warm-UP

If \( \frac{3x}{2} - 4 \cdot \frac{x}{2} \), then \( x = ? \)

A. \( \frac{1}{4} \)  
B. 1  
C. 2  
D. 4  
E. 5

Illuminations: Computer Animation

Notes from discussion of computer animation

How can we use a matrix to work with vectors?

Which transformations produce figures congruent to the original?
Calculating area with a Determinant

Use two vectors, \( \mathbf{u} = \langle 3, -1 \rangle \) and \( \mathbf{v} = \langle 6, 2 \rangle \), to create a parallelogram. As a group, determine ways to find the area of the parallelogram.

Determinant of a 2x2 matrix.

Matrix work on Calculators
Notes from class discussion

Practice
1. Write two sentences recalling something you learned from the computer animation activity.

2. What types of operations were used to transform your plane figure?

3. If a matrix has a determinant of -3, what effect will this matrix have on the transformation of a plane figure?

4. Given \( A = \begin{bmatrix} 3 & 0 \\ 4 & -1 \end{bmatrix} \) and \( B = \begin{bmatrix} -3 & 2 \\ 1 & 8 \end{bmatrix} \),
   
a. find \( 4A \).

   b. find \( A + B \).

   c. find the determinant of \( A \).

   d. Draw matrix \( B \) as two vectors in the coordinate system.

5. Do all matrix transformations result in congruent figures? Explain your response.

ACT Practice
If \( 9k - 3 = 3k + 15 \), then \( k = ? \)
Use precise vocabulary to describe what you see in the Eiffel Tower.
Construction Tools

Copy an arc

Write two sentences describing what you needed to complete this construction.

Add notes from class discussion.
Write two sentences to explain how the group used the tools to copy a segment.
Bisect a Segment

Write two sentences describing what you needed to do to complete this construction.

Add notes from class discussion.
Copy an Angle

Write two sentences describing what you needed to do to complete this construction.

Add notes from class discussion.
Bisect an Angle

Write two sentences describing what you needed to do to complete this construction.

Add notes from class discussion.
1. Write two sentences that describe the structure of the Eiffel Tower. Use precise language.

2. Write two sentences to describe how a string and a straight edge can be used to make accurate constructions.

3. Write two sentences to compare the process of copying a segment with that of copying an angle.

4. Write one sentence to describe how a construction is like a transformation.

5. Given an explicit definition for an arithmetic sequence, \( f(n) = 22 + 3n \), find \( f(14) \).

6. Write two sentences to describe how the mean of a data set compares to the median.

7. Collect the ages of at least five people. Calculate the mean and find the median of the data set.

ACT Practice

The vertices of \( \triangle ABC \) are \( A(-2,4), B(0,-5) \), and \( C(2,4) \). What is the line of symmetry, if any, of \( \triangle ABC \)?
ACT Warm-Up

Which letter has rotational symmetry but NOT reflectional symmetry?

- A. A
- B. C
- C. O
- D. Z

Write one sentence to review how to use your string and straightedge as construction tools.

Perpendicular lines

Write two sentences describing what you needed to do to complete this construction.

Add notes from class discussion.
Constructing a line parallel to a given line through a point not on a line

Write two sentences describing what you needed to do to complete this construction.

Add notes from class discussion.
How would you construct a square?

Write two sentences describing what you needed to do to complete this construction.

Add notes from class discussion.
Use precise vocabulary to describe what you see in Spaceship Earth.
How can you construct the given triangle?

Notes from class discussion
Construct the given hexagon.

Notes from class discussion.
  Regular, equilateral?
1. Consider the construction below.

Use a ruler to draw a segment and its midpoint.  
Draw another segment so the midpoints coincide.  
Connect the endpoints of the segments.

Use this construction with a variety of starting segments. What type of figure does the construction produce?

Make a mathematical argument that explains why that figure is produced each time by the construction.

2. Given a table of values, use different representations to describe the sequence.

<table>
<thead>
<tr>
<th>Step</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

Mary is constructing the perpendicular bisector of $JK$. First, she places her compass on point $J$ and draws $FGH$. Mary plans to draw exactly one more arc. At what point must Mary place her compass to complete the next step in the construction?
ACT Warm-Up
Which of the following numbers are integers?
a) $\sqrt{2}$
b) $\frac{3}{2}$
c) $\frac{12}{3}$
d) $\sqrt{6}$
e) $\sqrt{81}$

You are in charge of creating all the roof trusses for the new garage. Your plan is to build each triangular truss, to be raised and installed at a later date. You have only been given the dimensions of the boards to be used. Is this enough information to make sure that all the trusses are congruent?

Additional notes from class discussion
You are in charge of building the Cadet Chapel.

You have designed plans for construction that define the lengths of the two sides and the angle formed by them. Without any other information, can you be assured that your construction crew will create congruent triangles for the structure?

Notes from class discussion
You are in charge of spotlighting the glitter ball hanging from the center of the gym ceiling. Sets of two spotlights come connected with a 30 foot cord between them. Each spotlight is set at the same angle. You are considering placing the sets of spotlights in a circle on the floor but are unsure about whether or not they will all strike the glitter ball. Will this setup work? Discuss, using precise vocabulary.

List some requirements in the production of congruent triangles.
Is it enough information to have two sets of angle measures specified?
1. Define congruent.

2. To save time in construction, what are the minimum requirements in creating congruent triangles?

3. Write one sentence to describe the results of your construction work with sticks or straws today.

4. Decide whether or not the following represents a function. Explain your response. The vending machine produces soda pop when money is put in. On certain days, a person puts in fifty cents and gets one soda pop. On other days, a person puts in fifty cents and gets two soda pops. And still on other days, a person puts in fifty cents and gets nothing.

5. Graph a triangle with vertices at A(-4, -3), B(0,2), and C(-1, -2). Transform the triangle according to the following definition \( f(x, y) \rightarrow (-x, -y) \).

ACT Practice
Is the relation \( \{(0, 2), (1, 4), (0, 1), (3, 5)\} \) a function? Explain.
ACT Warm-Up

Which of the following numbers is composite?

1, 43, \( \frac{2}{3} \), 57, or 83

If you can, explain the meaning of PEMDAS:

Write one sentence to explain why might it be better to think of it like PEMDA?

Would PEDMSA be incorrect? Explain in one sentence.

Notes from class discussion.

What kind of symbols does P deal with other than parenthesis?

What else could E stand for other than exponents?
Order of Operations

Mathematical operations should *always* be performed in the following order:
1. Parentheses and absolute values
2. Exponents / Radicals
3. Multiplication and Division (from left to right as you encounter them)
4. Addition and Subtraction (from left to right as you encounter them)

Parentheses
Notation: ( ), [ ], grouping symbols (implied parentheses)

Examples:

- a. \(3(4 - 1) = \) ________
- b. \((3 - 2) + (6 + 1) = \) ________
- c. \(\frac{17 - 7}{3 + 2} = \) ________
- d. \(\frac{2 + 5 - (4 - 2)}{7 - 6} = \) ________

Practice.
1. Try example (b) above without the parentheses. Your equation should be
   \(3 - 2 - 6 + 1 = \) ____________. Do you get the same answer as the example? Why or why not?

Find the value of the following expressions:
2. \(2(6 + 3) - (7 + 5) = \) ____________

3. \(\frac{9}{(6 - 3)} = \) ____________

Write one sentence to explain the order of operations. Explain which parts may be tricky for you.
Write what you recall about absolute value.

Notes from class discussion.

Examples:  a.  |-8| = ______  b.  |5| = ______
   c.  |12-18| = ______  d.  5+ | 9 – 12 | = ______

Practice.
Find the value of the following expressions:
1.  | -7*3 | = ______  
   2.  \frac{8- | 2 - 9 |}{6} = _____

Exponents / Radicals

Remember to use implied parentheses with radicals.

So \( \sqrt{9 + 7} = \sqrt{9 + 7} = \sqrt{16} = 4 \)

Examples:  a.  \( 3^2 + 5 = \) ______  b.  \( \sqrt{5 + 4} = \) ______
   c.  \( (5 - 3)^3 + 2 = \) ______  d.  \( \sqrt{8*2} - 6 = \) ______
Practice.

Evaluate the following expressions:

1. \(30 - (2 + 3)^2 = \) _______

\[ \frac{5 + \sqrt{12 \times 3}}{3} = \] _______

Examples:

a. \(3[5 - (4 + 2)] - 3 = \) _______

b. \(19 - 2(7 - 3)^2 = \) _______

c. \(\sqrt{8 - 4} - 7^2 = \) _______

d. \(5(3 + 7) + 6(7 - 2)^2 = \) _______

Practice.

3. What is the value of \(7^2 - 2[3 + 2(5 - 1)]\)?

a. 9
b. 940
c. 27
d. -8
e. -9
Evaluate the following expressions. Please No calculators.

1. \(8 + 5 \times 3 = \) _____
2. \(12 \times 5 - 3 \times 6 = \) _____
3. \(5(6 - 2) = \) _____
4. \(6 - 2 \times 2 + 2^5 = \) _____
5. \(2 \times 5^2 = \) _____
6. \(\frac{3}{4} \times \frac{1}{2} + \frac{2}{3} = \) _____
7. \(\frac{8 - 5}{24 - 20} = \) _____
8. \(3[4 + 3(6 - 4)] = \) _____
9. \(\frac{4 \times 3 + 2}{4 + 3 \times 2} = \) _____
10. \(\frac{3 + 6(8 - 5)}{4^2 + 2} = \) _____
11. \(\frac{\sqrt{20 - 4}}{(2 + 3) - (5 - 2)} = \) _____
12. \(2[5 + 2(8 - 3)]^2 = \) _____
13. \(1 - |4 + 3| + 2 = \) _____
14. \(\frac{4 + |5 - 9| - |2 - 1|}{|4 - 7|} = \) _____

15. If the data point (3, -7) were reflected about the y-axis, what are the resulting coordinates?

16. Write two sentences defining an outlier of a data set.

ACT Practice. Evaluate \(-2 - [-3^2 + (-1)^3] = \) ?

A. -12  
B. 6  
C. 16  
D. -18  
E. 8
ACT Warm-Up

Evaluate $4^2 - 3 - 5 \cdot 8 - 2[(-3) - (-7)] = ?$

As a group, under each of the given words, write two similar words that you are familiar with. Then write a brief definition of the words that you chose.

<table>
<thead>
<tr>
<th>Associative</th>
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<table>
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<tr>
<th>Distributive</th>
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<tr>
<th>Commutative</th>
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<th>Identity</th>
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<th>Inverse</th>
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Make additional notes from class discussion.

_Illuminations: Algebraic Transformations_
Notes from class discussion
The Commutative Property of Addition

In symbols: For any real numbers $a$ and $b$,
$$a + b = b + a$$

1. Draw a diagram that shows this property.

The Commutative Property of Multiplication

For any real numbers $a$ and $b$,
$$a \cdot b = b \cdot a$$

2. The array shows a representation of the product $a \cdot b$.

How might this array also represent the product $b \cdot a$?

3. Explain why is there no Commutative Property of Subtraction or Commutative Property of Division? Illustrate with specific examples.

The Associative Property of Addition

Formally, for any real numbers $a, b$ and $c$,
$$(a + b) + c = a + (b + c)$$

1. Draw a diagram that illustrates this property.

The Associative Property of Multiplication
Distributive Property

In symbols: For any real numbers $a, b, \text{ and } c$

$$(a + b) \cdot c = a \cdot c + b \cdot c$$

Use the distributive property to write an equivalent expression.

2. $5(3 + 4)$

3. $x(3 + y)$

4. $x(x^2 + 3)$

5. $7(20 - 2)$

Another way to use the distributive property is to break up multiplication problems. For example $7 \cdot 23$ may be written as $7 \cdot 20 + 7 \cdot 3$, which equals $140 + 21$. Does this make the problem easier to do in your head?

Try these.

6. $8 \cdot 52$

7. $9 \cdot 84$

8. Substitute some values for $a, b, \text{ and } c$ for $\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$. Do you think this equation is true for all values of $a, b, \text{ and } c$?

Because of the relationship $a + c = \frac{a}{c} = a \cdot \frac{1}{c}$ division problems may be written as multiplication problems, and visa versa. Can you see why this would make the equation in #8 for all values?
The inverse property of addition

Formally, for any real number \( b \), there exist a number \(-b\) such that
\[
b + (-b) = 0
\]

The inverse property of multiplication

It tells us that for any real number \( b \), there is a number \( \frac{1}{b} \) such that
\[
b \cdot \frac{1}{b} = 1
\]
\( \frac{1}{b} \) and \( b \) are called *multiplicative inverses* or *reciprocals* of each other.

The identity property of addition

In symbols: For any real number \( a \),
\[
a + 0 = a
\]
For example: \( 48 + 23 \) can be written as \( 48 + 0 + 23 \). (I know, it seems silly, but wait!)
\[
48 + 0 + 23 =
\]
\[
48 + (2 + -2) + 23 =
\]
\[
(48 + 2) + (23 - 2) =
\]
\[
50 + 21 =
\]
\[
71
\]
1. Why is \( 27 + 14 \) the same as \( 30 + 11 \)?

Try these addition problems using the shortcut (add zero: take from one, give to the other).

2. \[
78 + 23
\]
3. \[
997 + 244
\]
The Identity Property of Multiplication

In symbols: For any real number $a$,

$$a \cdot 1 = a$$

Simplify: $\frac{3}{4}$ (Show two ways)

Discuss the equation $a + 0 = a \cdot 1$.

Matrix Properties
Practice Set 33

Match each equation with the property it illustrates.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>((5 + 3) + 7 = 5 + (3 + 7))</td>
<td>A. Commutative Property of Addition</td>
</tr>
<tr>
<td>(2 + 5 + (-5) = 2 + 0)</td>
<td>B. Identity Property of Addition</td>
</tr>
<tr>
<td>((5 + 0) \cdot (3 + 6) = (3 + 6) \cdot (5 + 0))</td>
<td>C. Associative Property of Multiplication</td>
</tr>
<tr>
<td>(\left(\frac{5}{7}\right) \left(\frac{3}{3}\right) = \frac{15}{21})</td>
<td>D. Commutative Property of Multiplication</td>
</tr>
<tr>
<td>(3 + (2 + 8) = 3 + (8 + 2))</td>
<td>E. Identity Property of Addition</td>
</tr>
<tr>
<td>(7 + 4 + 0 = 11)</td>
<td>F. Associative Property of Multiplication</td>
</tr>
<tr>
<td>(5(2) + 7(2) = (12)(2))</td>
<td>G. Inverse Property of Multiplication</td>
</tr>
<tr>
<td>(5 \cdot \frac{2}{5} \cdot \frac{5}{2} = 5 \cdot 1)</td>
<td>H. Inverse Property of Addition</td>
</tr>
</tbody>
</table>

9. Use the diagram below that shows a pattern consisting of trapezoids.

![Trapezoid Diagram](image)

a. Make as many observations as you can about the trapezoid pattern.
b. Find the perimeter of the first four trapezoid patterns shown above.
c. Find the perimeter of the pattern that contains 12 trapezoids without drawing a picture.
d. Write a generalization that can be used to find the perimeter of a pattern containing any number of trapezoids.
e. Using words, numbers and/or connections to the visual diagram, prove that the generalization you created in part 4 will always work.

ACT Practice

Which expression would be appropriate to complete the following equation in order for the equation to illustrate the identity property of addition: \(5 + (7 + 0) = ?\)

A. \((7 + 0) + 5\)  B. \(5 + (0 + 7)\)
C. \((5 + 7) + 0\)  D. \(5 + 7\)
E. \(12\)
ACT Warm-Up

Write one sentence to recall what inverse properties are.

Write one sentence to describe how inverses can help solve equations.

As a group, define each of the following terms:
Equation

Solution of an equation

Solve an equation

Isolating a variable

Make additional notes or corrections after class discussion.

Michelle plotted point $S$ on this graph. Which is a point in Quadrant IV that has an $x$-coordinate with the same absolute value as the $x$-coordinate of $S$?

A. (-4, 5)
B. (7, 6)
C. (7, -8)
D. (6, -5)
Is 5 a solution to the equation $2x - 7 = 3$? Explain:

Is $x$ isolated in the equation $x + 3 = 8$? Explain:

How do you know when an equation is solved?

Given that the same number is added to each of two equal quantities, will the resulting sums be equal?

Addition property of equality: For all real numbers $a$, $b$, and $c$ if $a = b$, then $a + c = b + c$.

Explain the Addition property of equality in your own words.

Write a short narrative that illustrates the Addition property of equality.

How can the addition property of equality be used to isolate $x$ in the equation $x + (-7) = 3$?

Group question: Technically two more properties of the real numbers are used to isolate $x$. Can you name them and how they were used?

Class discussion notes:
Partner Activity: Take turns solving the following problems. While a partner solves a problem the other partner will identify the property used for each step.

1. \( x + (-7) = 13 \)

   \[ \text{___________________________________________________} \]
   \[ \text{___________________________________________________} \]
   \[ \text{___________________________________________________} \]

2. \( x - 11 = 3 \)

   \[ \text{___________________________________________________} \]
   \[ \text{___________________________________________________} \]
   \[ \text{___________________________________________________} \]

3. \( x + 6 = 15 \)

   \[ \text{___________________________________________________} \]
   \[ \text{___________________________________________________} \]
   \[ \text{___________________________________________________} \]

4. \( x + 7 = 3 \)

   \[ \text{___________________________________________________} \]
   \[ \text{___________________________________________________} \]
   \[ \text{___________________________________________________} \]

Notes from class discussion
If two equal values are each multiplied by the same value will the resulting products also be equal? Explain.

Modify the addition property of equality (For all real numbers a, b, and c if a = b, then a + c = b + c.) so that it defines the multiplication property of equality.

**Multiplication Property of Equality:**

Fill in the blank to illustrate the multiplication property of equality.

If three feet are equal to one yard then ________________________________________.

If 4 cups are equal to 2 pints then ___________________________________________.

How might the multiplication property of equality be used to solve $\frac{1}{3}x = 4$? What other properties of the real numbers are necessary?

Rewrite $x \div \frac{4}{7}$ as an equivalent expression that involves multiplication.

What value would you multiply both sides of $x \cdot 15 = 4$ by to isolate $x$?

Groups: Solve $x \div 8 = 3$ Show and justify every step.
Partner Activity: Take turns solving the following problems. While a partner solves a problem the other partner will identify the property used for each step.

1. \( \frac{2}{5} \cdot x = 6 \)

2. \( x \cdot \frac{3}{7} = 3 \)

3. \( x + \frac{3}{4} = 6 \)

4. \( \frac{2x}{7} = 8 \)

Notes from class discussion.
Name That Property
The following problems have been solved. Justify or explain each step in the solution with: commutative property, associative property, identity property, inverse property, distributive property, subtract by adding the inverse, divide by multiplying by the inverse, or checking by substitution.

1. \( x - 8 = 17 \)
   \[ x + (-8) = 17 \]
   \[ x + (-8) + 8 = 17 + 8 \]
   \[ x + [(-8) + 8] = 25 \]
   \[ x + 0 = 25 \]
   \[ x = 25 \]
   \[ (25) - 8 = 17 \]

2. \( 7 + x = -9 \)
   \[ x + 7 = -9 \]
   \[ x + 7 + (-7) = -9 + (-7) \]
   \[ x + [7 + (-7)] = -16 \]
   \[ x + 0 = -16 \]
   \[ x = -16 \]
   \[ 7 + (-16) = -9 \]

3. \( 5x = 75 \)
   \[ \left( \frac{1}{5} \right) 5x = \left( \frac{1}{5} \right) 75 \]
   \[ \left[ \left( \frac{1}{5} \right) \cdot 5 \right] x = \left( \frac{1}{5} \right) 75 \]
   \[ 1 \cdot x = \left( \frac{1}{5} \right) \cdot 75 \]
   \[ x = \left( \frac{1}{5} \right) \cdot 75 \]
   \[ x = 75 \cdot \frac{1}{5} \]
   \[ x = 75 \div 5 = 15 \]
   \[ 5(15) = 75 \]

Practice with literal equations
Justify or explain each step in the solution with: **commutative property**, associative property, identity property, inverse property, distributive property, subtract by adding the inverse, divide by multiplying by the inverse, or checking by substitution.

1. \[ 7 + x = -1 \]
   \[ x + 7 = -1 \]
   \[ x + 7 + (-7) = -1 + (-7) \]
   \[ x + [7 + (-7)] = -8 \]
   \[ x + 0 = -8 \]
   \[ x = -8 \]
   \[ 7 + (-8) = -1 \]

2. \[ \frac{2}{3} x = 12 \]
   \[ \left(\frac{3}{2}\right) \cdot \frac{2}{3} x = \left(\frac{3}{2}\right) \cdot 12 \]
   \[ \left[\frac{3}{2} \cdot \frac{2}{3}\right] \cdot x = \left[\frac{3}{2} \cdot \frac{12}{2}\right] \]
   \[ 1 \cdot x = 18 \]
   \[ x = 18 \]
   \[ \frac{2}{3} \cdot (18) = \frac{2}{3} \cdot \frac{18}{3} = 2 \cdot 6 = 12 \]

Write the **first step** to solve each problem and the property that justifies this step.

3. \[ x + 23 = 4 \]
4. \[ x \cdot \frac{1}{8} = -2 \]
5. \[ x + (-7) = 50 \]

6. \[ n \cdot (-9) = 14 \]
7. \[ 3 - d = 15 \]
8. \[ \frac{t}{31} = -2 \]
Show and justify each step as you solve the following equations. Be sure to check your answer.

9. \( x + (-4) = 16 \)  

10. \( \frac{1}{5} = 4 \)

11. \( x + 6 = -11 \)  

12. \( 7 - x = 18 \)

13. \( \frac{n}{-4} = 13 \)  

14. \( b + 12 = -2 \)

15. Given the data set of the number of students with perfect attendance each term, find the mean and standard deviation. Explain your results.

ACT Practice.

What is the value of the expression \( 2y[(3x)^2 - 5x + 12] \) when \( x = 4 \) and \( y = 5 \)?
ACT Warm-Up

Which expression is equivalent to \((x + 8)(x - 7)\) ?

A. \((7 - x)(8 + x)\)  
B. \((8 + x)(7 - x)\)  
C. \(-(7 - x)(x + 8)\)  
D. \(-7(x + 8)(x + 8)\)

Write two sentences that describe how comfortable you are using properties to justify your work.

As a group, use multiple representations to describe the situation.
You start with $42 on your lunch balance, and are charged $2.50 everyday you eat at the cafeteria.

Is the lunch balance a function of the number of days you eat in the cafeteria? Explain.

What is the domain and range?

What is the slope of the data?

What is the vertical intercept and what is its meaning?

How many meals can you get before you run out of money?

How much money will be left in the balance?
As a group, use multiple representations to describe the situation. Anna has 20 gallons in the tank of her Toyota Tercel, which uses 1/35 gallon per mile (35 mpg). She has to drive a great many miles to visit a possible college.

Is the amount of gas in the tank a function of the number of miles she drives? Explain.

What is the domain and range?

What is the slope of the data?

What is the vertical intercept and what is its meaning?

How many miles will she be able to travel on the 20 gallons of fuel?

Find the ordered pair at the horizontal intercept of the graph. Explain the meaning.
As a group, analyze the following situation. Phillip rides an elevator that starts on the 17th floor and goes downward 1 floor in 3 seconds (or at the rate of 1/3 floor per second). Use different representations to describe the number of floors that Phillip is above ground level after a given number of seconds.

Is the number of floors that Phillip is above ground level a function of the number of seconds that tick by? Explain.

What is the domain and range?

What is the slope of the data?

What is the vertical intercept and what is its meaning?

How long will it take Phillip to reach the ground level?

Find the ordered pair at the horizontal intercept of the graph.
As a group, analyze the following situation.
Rebecca knows that she can average 43 mph (.724 miles per minute) from her grandma’s house in Mapleton to her home 38 miles away in Heber City. Use different representations to describe the number of miles Rebecca is away from home given the number of minutes traveling there.

Is the number of miles from home a function of the number of minutes traveled?
Explain.

What is the domain and range?

What is the slope of the data?

What is the *vertical* intercept and what is its meaning?

How long will it take Rebecca to get home?

Find the ordered pair at the horizontal intercept of the graph.
As a class, analyze the following situation.
Andy wants to drop weight for wrestling. He weighs 187 now, and plans to lose 1.5 lbs. per week. Use different representations to describe the situation.

Write an equation to find the number of weeks until Andy weighs 178 lbs then solve the equation showing and justifying every step.
As a group, analyze the following situation. A hot-air balloon starts at 1000 feet above Heber City and rises at a rate of 5 feet per second. Use different representations to describe the situation.

Write an equation to find when the balloon reaches 3200 feet then solve the equation showing every step.
Practice analyzing the situation.

On August 5, 2010, 33 miners were trapped 2300 feet underground. After 69 days the men were brought to the surface one at a time in the pod shown below. The pod could be pulled up as fast as 92 feet per minute.

Use different representations to describe the pod’s height from the bottom of the shaft at a given number of minutes of a miner’s rescue.

Write and solve an equation to find the number of minutes for the pod to reach the surface. Show and justify every step.
1. Analyze the following situation. You know that you can mow 20 square yards of lawn in 1 minute, and that your whole yard has 1600 square yards of lawn. Use different representations to describe the number of yards left to mow given the number of minutes that you have been mowing the lawn.

Is the number of yards left to mow a function of the number of minutes spent mowing? Explain.

What is the domain and range?

What is the slope?

How long will it take to mow the whole lawn?

Find the ordered pair at the horizontal intercept of the graph.

Write one sentence to compare the horizontal and vertical intercept.

(Continued)
2. Miguel knows that it will take $6000 dollars to pay for his first year of college, so he set aside the $1200 that he earned this summer in a savings account then plans to deposit $200 per month to the account. Use different representations to describe Miguel’s college savings for a given number of months of deposits.

Write and solve an equation to find the number of months for Miquel to reach his goal. Show and justify every step.

3. Given a function $f(n) = 3f(n - 1)$, with $f(0) = 1$, create a data table with the first six terms. Create an explicit formula for the sequence.

ACT Practice.

Which expression is the opposite of $2(3x - 8)$?

A. $2(3x + 8)$  
B. $2(8 - 3x)$  
C. $\frac{1}{2(3x - 8)}$  
D. $-\frac{1}{2(3x - 8)}$
Solve. Justify each step.

$5 - 2(x + 5) = 12$

Extra Practice.

What property is shown by $5 \cdot \left(\frac{1}{5}\right) = 1$? By $1 \cdot x = x$?

Group investigation:
Simplify.

$12\left(\frac{2}{3}\right)$  $12\left(\frac{3}{4}\right)$  $12\left(1 - \frac{5}{6}\right)$  $12\left(\frac{1}{2} + \frac{5}{12}\right)$

What happened to the fractions? Why?

Make additional notes or corrections from class discussion.
Write one sentence to describe your reaction on seeing the following problems.

Solve  \[ \frac{3}{4}x - \frac{1}{12} = \frac{x}{6} + \frac{1}{3} \quad 1.5x - .12 = .63 \]

Solve and justify your steps.

Practice. What number could you multiply both sides of \[ \frac{1}{2}x + \frac{4}{5} - 0.9 = 0.2 \] to clear the fractions and decimals? Explain.

Solve.
1. Dividing both sides of an equation by \( \frac{5}{6} \) is the same as

2. What number could you multiply both sides of \( \frac{1}{2}x + \frac{3}{5} - 0.7 = 0.3 \) to clear the fractions and decimals? Explain.

3. What is the first step to solve \( 4(3x - 2) + x = 7 \)?

Solve the following equations. Justify your steps. Be sure to check your answers.

4. \( \frac{2}{3}x - 5 = 19 \)

5. \( 6\left(\frac{1}{3}x - \frac{1}{2}\right) + x = 5 \)

6. \( 3x + 5 = 6x - 2 \)

7. \( 1.2x - .5 = -1.3 \)

8. \( \frac{1}{3}x + \frac{5}{6} = 2 \)

9. \( 3 - 2(x - 7) = x + 3 \)

(Continued)
10. Mel can paint 5 cars in 3 days. So far he has painted 6 cars. If Mel has to paint 21 cars altogether, the equation \( \frac{5}{3}t + 6 = 21 \) can be solved to find the remaining number of days to finish the job. How many days will it take him?

11. Given a domain of a circle whose center is at (-1,5) and radius is 3, Graph the circle.

On the same graph, illustrate the following function.

\( f(x, y) \rightarrow (x - 3, y - 2) \)

ACT Practice.

This table shows the relation between the number of calories burned, \( c \), and the time in minutes, \( t \), Tyrone walked at a constant speed on a treadmill.

<table>
<thead>
<tr>
<th>( c )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>5</td>
</tr>
<tr>
<td>85</td>
<td>10</td>
</tr>
<tr>
<td>130</td>
<td>15</td>
</tr>
</tbody>
</table>

If Tyrone continues walking at the same speed, how many calories will he burn in 25 minutes?
ACT Warm-Up  Which of the following is a rational number?
A.  .1434562321
B.  $\sqrt{9}$
C.  0.1121231234…
D.  $\sqrt{-4}$
E.  $\sqrt{5}$

Class: Define cost and revenue functions

Barry plans to start a fly-tying business in an unused room in the basement of his parents’ house. He wants to build four work stations at a cost of $1200 and he knows that each fly will have an average cost of $0.75 to make. Define appropriate variables then create a formula that finds cost as a function of flies.

He knows that he can sell the flies to a local shop for $1.10 a piece. Define appropriate variables then create a formula that can be used to calculate Barry’s revenue (the amount of money coming in).

Groups: Complete the following table using the cost and revenue functions that were defined by the class. Make sure to write the cost and revenue functions under each of their headings.

<table>
<thead>
<tr>
<th>Flies</th>
<th>Cost</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Write a sentence to describe each sequence of data.

Graph the data for the cost together with the data from the revenue. Make sure to label your graph carefully.

Use your graph to estimate when Barry’s business will break even.
The symbol $>$ means “greater than”.
Whenever you see the symbol $>$, think ____________________________

The symbol $<$ means “less than”
Whenever you see the symbol $<$, think ____________________________

Equations always contain the symbol $=$, which means

**Inequalities** contain the symbols $<, >, \leq, \geq$, or $\neq$.

The symbol $\leq$ means ____________________________________________

The symbol $\geq$ means ____________________________________________

The symbol $\neq$ means ____________________________________________

Use the variables that the class defined for Barry’s business formulas to write the following in symbols.

1. The revenue exceeds cost. ___________________ or ___________________
2. The amounts of revenue and cost are not the same. ___________________
3. The revenue is at least the cost. _________________ or ________________
4. The cost exceeds the revenue by more than $1000.
   _________________ or _________________
5. Barry would have to increase his revenue by at least $2500 to cover costs.
   _________________ or _________________
6. Write an inequality in terms of the **number of flies** that means revenue exceeds cost.

7. *Profit* is the difference between revenue and cost. Suppose we define the variable $p$ to represent Barry’s profit. In terms of cost and revenue write inequalities that make the following true:

   $p < 0$ ____________________________ $p \geq 0$ ____________________________
The statement: “I need to score at least a twenty on my next exam” can be written algebraically as the inequality
\[ S \geq 20 \]
The inequality could be graphed.

Another statement: “There are only thirty points possible on the exam” can be written algebraically as the inequality
\[ S \leq 30 \]
This inequality could also be graphed.

The statement: “I need to get at least a score of 20 on a 30 point exam” can be written algebraically as the inequality
\[ 20 \leq S \leq 30 \]
A combination of inequalities is like this is called a *compound inequality*. It means the score is greater than or equal to 20 and less than or equal to 30. This particular type of compound inequality is called an *intersection* because it is where graphs cross over each other.
The graph of \( 20 \leq S \leq 30 \) looks like this:

How would the graph of \( 20 \leq S \leq 30 \) be different if the 30 questions of the exam were graded as right or wrong with no partial credit? Show the graph.

Write an inequality for the statement: “Tommy will spend at least $10 on gift for Gina.”

*Inequality* ______________________________

*Graph* ________________________________
Write an inequality for each statement

Sadie wants to run the 5K in less than 21 minutes.
Inequality___________________________
Graph

The legal freeway speed is between 45mph and 65mph.
Inequality___________________________
Graph

Jonah can hold his breath for no more than 132 seconds.
Inequality___________________________
Graph

Often inequalities must be solved like equations.

“The difference between the actual part measurement (x) and the ideal part measurement of 3 inches can be no more than $\frac{1}{16}$ inch.”

Inequality $-\frac{1}{16} \leq x - 3 \leq \frac{1}{16}$ can be solved by adding 3 to each number.

$-\frac{1}{16} \leq x - 3 \leq \frac{1}{16}$ to get $2 \frac{15}{16} \leq x \leq 3 \frac{1}{16}$

$+3 \quad +3 \quad +3$

The graph shows the acceptable measurements for the part.
1. Write two different inequalities to compare the two quantities. Gina is 14 and Bill is 13.

2. Write an inequality to describe the following situation. Juan pays $52.35 a month for his cable bill and an additional $1.99 for each streamed movie. To stay in budget, he needs to keep his total bill below $75.

3. Draw a graph for each inequality.

   - \[ x \leq 6 \]
   
   - \[ k \leq -2 \]

4. Write an inequality for each graph.

ACT Practice
Which of the following is a rational number?
A. \( \pi \)
B. \( \sqrt{8} \)
C. .1234...
D. \( \sqrt{23} \)
E. \( \sqrt{-9} \)
ACT Warm-Up
Evaluate. \(-3 - \left[ -2^2 + (-2)^2 \right] \)

As a group, analyze the following situation.
Craig is delivering boxes of paper to each floor of an office building. Each box weighs 64 pounds, and Craig weighs 160 pounds. If the maximum capacity of the elevator is 2000 pounds, how many boxes can Craig safely take on each elevator trip? Use different representations to describe this situation.

Notes from class discussion.
Building on what we know: **The steps for solving an inequality are very similar to solving an equation.** There is one big difference to look out for.

If \( a > b \) then \(-a < -b\).

This means that you must **reverse** the inequality symbol whenever you Multiply or Divide both sides by a negative value.

For example: \( 7 > 5 \) but \((-3)7 < (-3)5 \) since \(-21 < -35\)

---

**Solving an Equation**

\[
5 - 2x = 4
\]

Subtract 5 from both sides:

\[
-2x = -1
\]

Divide both sides by -2:

\[
x = \frac{1}{2}
\]

---

**Solving an Inequality**

\[
5 - 2x < 4
\]

Subtract 5 from both sides:

\[
-2x < -1
\]

Divide both sides by -2 **and reverse the inequality symbol**:

\[
x > \frac{1}{2}
\]

---

Remember subtracting is just adding the opposite, and dividing is just multiplying by the reciprocal.

**Rules for solving inequalities**: Use the same rules for equations, but remember to reverse the inequality symbol when multiplying (or dividing) by a negative value.

### Properties of Equality

- If \( a = b \), then \( a + c = b + c \)

### Properties of Inequalities

- If \( a > b \), then \( a + c > b + c \)

---

### Example

\[
\text{If } a = b, \text{ then } a \cdot c = b \cdot c
\]

### Example

\[
\text{If } a > b, \text{ and } c > 0, \text{ then } a \cdot c > b \cdot c
\]

### Example

\[
\text{If } a > b, \text{ and } c < 0, \text{ then } a \cdot c < b \cdot c
\]

---

### Examples

---
Practice. Justify each step as you solve the inequalities.

\[ -3 > -3 + k \]

\[ -3 \leq x + 5 \]

\[ 24 \leq p + 7 \]

\[ k + 20 < 1 \]

\[ -4 \geq \frac{n}{2} \]
As a group, analyze the following situation.

The snow leopard is one of many species of mammals on the endangered list. Imagine that a wildlife biologist is nursing a sick snow leopard back to health. The animal currently weighs 43kg.

A healthy leopard show weigh more than 70kg. The biologist estimates that a special diet will allow the cat to gain an average of 0.5kg per week. Write and solve an inequality to determine when the snow leopard’s weight will exceed 70kg.

Notes from class discussion.
1. Write one sentence to describe how solving inequalities can be different from solving equations.

2. Solve each inequality and justify each step.

\[ 16 + a \leq 10 \]

\[ \frac{b}{2} > -2 \]

3. Alida has at most $22.50 to spend at a convenience store. She buys a bag of potato chips and a bottle of soda pop for $1.75. If gasoline at this store costs $3.85 per gallon, how many gallons of gasoline can Alida buy for her car, to the nearest tenth of a gallon? Write an inequality for the situation and solve.

4. Given the two-way frequency table of favorite leisure activities, answer the following questions.

<table>
<thead>
<tr>
<th>Dance</th>
<th>Sports</th>
<th>TV</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>2</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Women</td>
<td>16</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>18</td>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

a. What percentage of men enjoy watching TV?

b. What percentage of dance enthusiasts are female?

c. Write one sentence to describe any trends that you see in this data table.

ACT Practice. Evaluate without a calculator. \[ \frac{2}{7} \div \frac{13}{14} \]
ACT Warm-Up

Which expression is equivalent to 
$2x + 3y - 5x^2 - 10y + 7x^2$ ?

A. $4x^2 - 7y$
B. $2x^2 + 2x - 7y$
C. $2x^2 - 5xy$
D. $2x + 13y + 12x^2$

Illuminations: Escape from the Tomb

Notes from class discussion of Escape from the Tomb lab
1. Recall in the Escape from the Tomb lab, you tracked the height of two different bowls with the *same type of weight* first. Write two sentences to compare those data sets.

2. Recall in the Escape from the Tomb lab, you then tracked the height of two different bowls using *different types of weights*. Write two sentences to compare those data sets.

3. What is the rate of change for the following two data sets? Will these data sets ever reach the same function value at the same time? Explain.

<table>
<thead>
<tr>
<th>n</th>
<th>f(n)</th>
<th>n</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>23</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>26</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>2</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>32</td>
<td>3</td>
<td>24</td>
</tr>
</tbody>
</table>

4. What is the rate of change for the following two data sets? Will these data sets ever reach the same function value at the same time? Explain.

<table>
<thead>
<tr>
<th>n</th>
<th>f(n)</th>
<th>n</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
<td>53</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>2</td>
<td>37</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>3</td>
<td>29</td>
</tr>
</tbody>
</table>

5. Given the definition $f(n) = 2f(n-1) + 7$ with $f(0) = 9$, create a data table with the first six terms of the sequence.

ACT Practice.
Solve for $a$. \( \frac{1}{2}(a-1) - \frac{1}{3}(2-a) = -1 \)
ACT Warm-Up

When the sum of some number and 5 is multiplied by 7 you get a value of 28. Find the number.

With your group, use different representations, including technology, to graph and compare a beginning salary of $30 the first day, increased by $5 each day and a beginning salary of $0.01 the first day, which doubles each day. When are the salaries equal? How do you know?
Notes from class discussion.
Consider the following two functions.

\[ f(x) = 295 - 15x \quad \text{and} \quad g(x) = -15x + 175. \]

With your group, use different representations, including technology, to graph and compare. When are the functions equal? How do you know?

Notes from class discussion.

Write two sentences to show your understanding or your questions about systems of equations so far.
Substitution Method of solving systems algebraically.

As a class, solve each system.

\[
\begin{align*}
y &= 7x - 10 \\
y &= -3
\end{align*}
\]

\[
\begin{align*}
y &= 6x \\
y &= 5x + 7
\end{align*}
\]

\[
\begin{align*}
y &= -4 \\
y &= x - 8
\end{align*}
\]

What does it mean to solve a system of equations?
1. In your own words, what does it mean to solve a system of equations?

2. Given two functions: \( f(x) \) with a rate of change of 17, and \( g(x) \) with a rate of change of 17, write two sentences to describe possible solutions to a system of these two equations.

3. Solve each system. Justify your method.

\[
\begin{align*}
y &= -8 \\
y &= -2x - 12
\end{align*}
\]

\[
\begin{align*}
y &= 9x - 9 \\
y &= 9
\end{align*}
\]

4. Solve the inequality and justify your work.

\[
14 - 2x > 12
\]

ACT Practice:

A public opinion survey explored the relationship between age and support for increasing the minimum wage. The results are summarized in the two-way table to the right.

In the 21 to 40 age group, what percentage supports increasing the minimum wage?

(A) 12.5%
(B) 20%
(C) 25%
(D) 50%
(E) 75%
ACT Warm-Up

Find the image of the translation of \((2,5)\) after a vertical shift of 3 and a horizontal shift of \(-4\).

As a class, consider the following situation.

For her book club, Courtney wants to have crackers and cheese on hand. She has $45 to buy mozzarella and gouda cheese. Courtney already has crackers. She can buy \(M\) pounds of mozzarella at $5 per pound or \(G\) pounds of gouda at $8 per pound. If Courtney spends all $45, the equation

\[5M + 8G = 45\]

describes all of the possible weight combinations of mozzarella and gouda that she could buy.

Use the equation \(5M + 8G = 45\) to answer the following questions.

1. How many pounds of gouda can Courtney buy if she buys 9 pounds of mozzarella?

2. How many pounds of mozzarella can she get if she buys 3 pounds of gouda?

3. How many pounds of gouda can Courtney buy if she buys 2 pounds of mozzarella?

4. How many pounds of gouda can she buy if she buys 0 pounds of mozzarella?

Use the data you collected in problems 1-4 to graph \(5M + 8G = 45\).
In your group, consider the following system of equations. Decide whether it is possible to solve this system. Explain.

\[ 5x + y = 9 \]
\[ 10x - 7y = -18 \]

Notes from class discussion.

Write two sentences to describe the elimination method and why it works.
As a group, consider the following situation.
The school that Stefan goes to is selling tickets to a choral performance but cannot remember the ticket prices from the previous year. The school accountant does, however, have information about past ticket sales and money totals. On the first day of ticket sales, the school sold 3 senior citizen tickets and 1 child ticket for a total of $38. The school took in $52 on the second day by selling 3 senior citizen ticket and 2 child tickets. Use different representations to find the price of a senior citizen ticket and the price of a child ticket.
Notes from class discussion.
1. What does it mean to solve a system of equations?

2. If two functions have different rates of change, will the system of those two functions have a solution? Explain your response.

3. Given the system, choose a method to solve. Justify your process.

\[
\begin{align*}
6x - 5y &= 22 \\
y &= -8
\end{align*}
\]

4. Brenda's school is selling tickets to a spring musical. On the first day of ticket sales the school sold 3 senior citizen tickets and 9 child tickets for a total of $75. The school took in $67 on the second day by selling 8 senior citizen tickets and 5 child tickets. What is the price each of one senior citizen ticket and one child ticket?

5. Describe each of the following sequence as arithmetic, geometric, or neither. Explain your response.

<table>
<thead>
<tr>
<th>n</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>n</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>n</th>
<th>f(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>31</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
</tr>
</tbody>
</table>

ACT Practice.
Find the image of the reflection of \((-4, 7)\) about the y-axis.
ACT Warm-Up
Bill’s scores on three exams were 85, 94, and 89. If the final exam in the class counts as much as two exams, what must he score on the final to get a 90 average in the class?

As a group, consider the following situation.
Juan pays $52.35 a month for his cable bill and an additional $1.99 for each streamed movie. Gail pays $40.32 a month for her cable bill and an additional $2.49 for each streamed movie. Who has the better deal? Use different representations to justify your choice.

Notes from class discussion.
What method of solving systems do you prefer? Does your response change depending on the format of the functions?

Practice solving systems. Justify your work.

1. \( y = 2x + 5 \)
   \( 3x - y = -4 \)

2. \( 3x - y = 4 \)
   \( 2x + y = 7 \)

3. \( 2x - y = -4 \)
   \( x + 3y = 5 \)

4. \( y = \frac{2}{3}x + 4 \)
   \( 2x - 3y = -12 \)
Using matrices to solve systems

Why is an inverse of a matrix necessary?

How do we know if a matrix has an inverse?
Practice

\[ x - 5y = -5 \]
\[ -4x - 2y = 20 \]

\[ 2x + 2y = 0 \]
\[ 4x - y = -20 \]

\[ -x - y = -1 \]
\[ 3x + 3y = 3 \]

\[ -x + 4y = -2 \]
\[ -2x + 5y = -4 \]

\[ 4x + 4y = -32 \]
\[ 2x + 2y = -16 \]
RREF to solve larger systems

\[-2x - 5y + 4z = 21\]
\[-5x - 5y + z = 21\]
\[-4y - 4z = 8\]

\[-4x - 6z = -12\]
\[-6x - 4y - 2z = 6\]
\[-x + 2y + z = 9\]
1. How do you know if a matrix has an inverse?

2. When the calculator puts a matrix in reduced row echelon form, what solution method is it employing?


\[
\begin{align*}
3x - 4y &= 1 \\
-5x + 2y &= 3 \\
-5x + 5y &= 10 \\
-2x + 2y &= -4
\end{align*}
\]

4. Set up a matrix and use technology to solve.

\[
\begin{align*}
5x + y - 4z &= -4 \\
-3y - 6z &= -21 \\
-x - y - z &= -6
\end{align*}
\]
ACT Warm-Up
Find the cost of a $24 shirt if it is discounted by 20%.

**Exploration**

In Roll-a-rama, players roll one white die and one red die. One roll of each die is considered a game. To win, the following must be true:

- the number on the red die must be at least 2 but no more than 5
- the sum of the two dice must be less than or equal to 7.

Play Roll-a-rama 10 times and record your results.

List all the possible rolls in Roll-a-rama

If all the possible rolls were graphed on a coordinate plane where the roll on one die is represented on the x-axis and the roll of the other die is represented on the y-axis, what would the graph look like?
A feasible region is the graph of the solution set of a system of linear inequalities.

For example, consider the following system of linear inequalities: $0 \leq x \leq 6$, $0 \leq y \leq 4$, and $x + y \leq 8$. The shaded region in Figure 13-10 is the feasible region for this system. All the points in the feasible region are solutions to all the inequalities that define it.

Figure 13-10
A feasible region.

---

d. The points on the boundary lines of the feasible region in Figure 13-10 are part of the feasible region.

1. How do the inequalities that define the feasible region indicate that this is true?

2. How does the graph of the feasible region indicate that points on the boundary are included?

3. How would you change the graph if boundary points were not included in the feasible region?

e. The winning combinations for Roll-a-rama could be graphed as a feasible region.

1. Why might it be misleading to shade the feasible region for Roll-a-rama?

2. How might you represent the feasible region for Roll-a-rama?

Write two sentences to compare your previous work with systems of equations to systems of inequalities.
Solutions to systems of inequalities:

1. \[y \geq 2x - 3\]
   \[y < -x + 2\]

2. \[y \geq \frac{3}{4}x + 1\]
   \[y \leq \frac{3}{4}x - 2\]
3. \[ \begin{align*}
2x - y & \geq -1 \\
x + y & \leq 4 \\
x + 4y & \geq 4
\end{align*} \]
In example 3, we had a shaded region that was a polygon. It had corners or _________.
*These corners are important points!!!

How can we find the coordinates of those points?

We solve little ___________ of equations.

Let’s find the corners/vertices in example 3.
As a group, consider the following situation.
A landscaping company has crews who mow lawns and prune shrubbery. The company schedules 1 hour for mowing jobs and 3 hours for pruning jobs. Each crew is scheduled for no more than 2 pruning jobs per day. Each crew’s schedule is set up for a maximum of 9 hours per day. Use different representations to describe this situation.

On the average, the charge for mowing a lawn is $40 and the charge for pruning shrubbery is $120. Find a combination of mowing lawns and pruning shrubs that will maximize the income the company receives per day for one of its crews.
1. Write one sentence to describe how a system of linear inequalities is different from a system of equations.

2. Solve the system of inequalities.

\[
\begin{align*}
y & \leq \frac{5}{2}x - 2 \\
y & \geq \frac{1}{2}x + 2
\end{align*}
\]

3. Choose a method to solve the system of equations. Justify your work.

\[
\begin{align*}
7x + 2y &= 24 \\
8x + 2y &= 30
\end{align*}
\]

4. Write one sentence to compare recursive and explicit definitions of sequences.

5. The maximum and/or minimum function values will come from which points of a feasible region?

ACT Practice

\[
\frac{27^5}{9^4} = ?
\]
ACT Warm-Up
Solve $2 - 3x \leq 14$

As a group, consider the following situation.
As a receptionist for a vet, one of Dolores Alvarez’s tasks is to schedule appointments. She allots 20 minutes for a routine visit and 40 minutes for a surgery. The vet cannot do more than 6 surgeries per day. The office has 7 hours available for appointments. If an office visit costs $55 and most surgeries cost $125, find a combination of office visits and surgeries that will maximize the income the vet practice receives per day.

Notes from class discussion.
Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

\[
\begin{align*}
  y & \geq 1 \\
  x & \leq 6 \\
  y & \leq 2x + 1 \\
  f(x, y) &= x + y
\end{align*}
\]

Find one data point that satisfies the first inequality, but not the other two.

Find one data point that satisfies all three inequalities.

Find one data point that satisfies none of the inequalities.
Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

\[ \begin{align*}
2x + 2y &\geq 4 \\
2y &\geq 3x - 6 \\
4y &\leq x + 8 \\
f(x, y) &= 3y + x
\end{align*} \]

Find one data point that satisfies the first inequality, but not the other two.

Find one data point that satisfies all three inequalities.

Find one data point that satisfies none of the inequalities.
1. Explain the shading that has been part of the systems of inequality problems we have done in class.

2. Solve the system of inequalities.
   \[
   \begin{align*}
   y &< -3 \\
   y &\geq 4x + 1
   \end{align*}
   \]

3. Given the vertices of a feasible region (3,5), (-2, 7), and (16, 1), find the maximum value of the function \( f(x, y) = -2x + 5y \).

4. Write a function that translates a point \((x, y)\), three spaces down and four spaces right.

5. Write a function that rotates a point \((x, y)\), 90° about the origin.

ACT Practice.
A rectangle has a length of \( x - 3 \) and a width of \( 2x + 1 \). If the perimeter is 26, what is the value of \( x \)?
ACT Warm-Up
A rectangle has a length of \( x - 5 \) and a width of \( 3x + 2 \). If the perimeter is 34, what is the value of \( x \)?

Using technology to analyze data sets.

Choose a set of data from the following site:

Cheese: taste score vs. lactic acid

Use technology to create a Scatter Plot

Correlation

Regression
Correlation coefficient

Causation

As a group, study the following data.


<table>
<thead>
<tr>
<th>Year</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>9867</td>
</tr>
<tr>
<td>1980</td>
<td>21,023</td>
</tr>
<tr>
<td>1985</td>
<td>27,735</td>
</tr>
<tr>
<td>1990</td>
<td>35,353</td>
</tr>
<tr>
<td>1995</td>
<td>40,611</td>
</tr>
<tr>
<td>1998</td>
<td>46,737</td>
</tr>
</tbody>
</table>

Use technology to create a scatter plot. Analyze the data and the scatter plot to determine a function that could model the data. Use regression to create a model. Describe this model using the following vocabulary: linear/exponential, regression, correlation, and causation.

Use your model to make a prediction of the median income for the year 2012.

Notes from class discussion
As a group, choose a set of data from the following:

Hunting in New Guinea page

Use technology to create a scatter plot. Analyze the data and the scatter plot to determine a function that could model the data. Use regression to create a model. Describe this model using the following vocabulary: linear/exponential, regression, correlation, and causation.
1. When given a scatter plot of data, write two sentence to describe how you will choose from different regression forms (linear, exponential, something else).

2. If a set of data has a correlation coefficient of 0.3, what could you say about the regression equation?

3. Give an example of a data set that has strong correlation but no causation and describe why this is so.

4. Give an example of a data set that has both strong correlation and causation and write a description of why this is so.

5. The following data shows the age and average daily energy requirements for male children and teens.

<table>
<thead>
<tr>
<th>Age</th>
<th>Daily Energy (kcal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1110</td>
</tr>
<tr>
<td>2</td>
<td>1300</td>
</tr>
<tr>
<td>5</td>
<td>1800</td>
</tr>
<tr>
<td>11</td>
<td>2500</td>
</tr>
<tr>
<td>14</td>
<td>2800</td>
</tr>
<tr>
<td>17</td>
<td>3000</td>
</tr>
</tbody>
</table>

Use technology to create a scatter plot and find a function to fit the data. Use your function to find the daily energy requirement for a fifteen year old male. Would your model apply to an adult?

ACT Practice
How far is the point \((-3, 2)\) from the point \((5, -4)\)?
ACT Warm-Up
What percent of 4 is 12?

Illuminations: Growth Rate

Notes from class discussion

Write two sentences to describe a method of approximating growth rates for non-linear data sets.