B Groups: Relationships between measures of segments (chords, chord parts, secant segments, tangent segments)

The word **chord** comes from the word *chorde* meaning “gut” or “string,” (perhaps of a harp).

The word **secant** comes from the word *secare* meaning “to cut.”

The word **tangent** comes from the word *tangere* meaning “to touch.”

Describe why you think each of these segments/lines was named as they were.

You will be investigating segment **lengths** formed by these segments and lines.

In the following figure, describe lines CE and CG.  *(1 on GSP)*

Determine the lengths of $\overline{CD}, \overline{CE}, \overline{CF}, \overline{CG}$.  

![Diagram with labeled segments and lines]
Triangles CDG and CFE are similar (CFE is reflected over \(CD\), rotated, and dilated). The following figure shows the result after these transformations.

Determine the scale factor using two different ratios:

\[
\frac{CD}{CF'} \quad \text{and} \quad \frac{CG}{CE'}
\]

These scale factors should be the same!

Rewrite the equation \(\frac{CD}{CF'} = \frac{CG}{CE'}\) by first multiplying both sides by \(CF'\).

Multiply both sides of the equation by \(CE'\).

Write the result of your manipulation.

This result is known as the secant segment rule.

If two secant segments share the same endpoint outside a circle, then the product of the length of the secant and its external segment is the same as the product of the length of the other secant and its external segment.

Write the secant segment relationship for the following diagram.
Determine the value of the variable in each problem.

Investigate the following figure. \((2\text{ on GSP})\)

- Determine a list of transformations that would map triangle DEF onto triangle CBF.

- If triangles DEF and CBF are similar, the secant segment relationship applies here, as
  
  - Determine the lengths CF, BF, DF, and EF.

  Use these lengths to check whether or not the secant segment relationship holds.
Use the following figure to write out the chord segment relationship.

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

Find the value of \( x \) in each of the examples below.
Investigate the following figure. *(3 on GSP)*

Determine the lengths AB, BC, CD.

Triangles ADC and DBC are similar. (reflection, rotation, and dilation). The following figure shows the result after these transformations.

To determine the scale factor we could use the ratios \( \frac{CD}{CA} \) or \( \frac{CB'}{CD} \). They should be the same.

Rewrite the equation \( \frac{CD}{CA} = \frac{CB'}{CD} \) by first multiplying both sides by CA.

Multiply both sides of the equation by CD.

Write the result of your manipulation.

This result is known as the secant tangent segment rule. If a secant and a tangent intersect at a point outside the circle, then the product of the length of the secant and its external segment is the same as the product of the length of the tangent segment and itself.

Write the secant tangent segment rule for the given figure.
Determine the value of the variable in each problem.

Write a conjecture about two segments that are tangent to a circle from the same external point, as shown.

Things to share with your classmates:
- definitions of chord, secant, and tangent and where their names came from
- secant segment rule and where it comes from
- chord segment rule
- secant and tangent segment rule
- tangent segments from same external point
- example homework problems